Teaching Mathematical Epidemiology at a Variety of Levels Using Multiple Representation Theory

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August 19, 2020

Part 1: Theory

1. Multiple representation theory
2. Some modeling activities
3. Narrow and broad views

Multiple representation theory

Multiple representation theory was introduced by Diaz-Eaton et al (PRIMUS, 2019). It is a variant of the "Rule of Four."

- Models have multiple representations:
  - Verbal
  - Visual
  - Symbolic
  - Numerical
  - Experiential

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  - Verbal
  - Visual
  - Symbolic
  - Numerical
  - Experiential
- **Modeling: making connections between representations.**

Some modeling activities

- Choosing questions
- Choosing outcomes to report
- Choosing assumptions
- Translating assumptions to mathematics
- Determining parameter values
- Doing graphical analysis
- Doing symbolic analysis
- Running simulations
- Presenting results
- Communicating results

Narrow and broad views

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- **Broad view**: A map from the space of parameters to the space of outcomes.
- **The technical work is in the narrow view, but most of the modeling is in the broad view.**
  - Example: Plot a graph of the final susceptible population as a function of the basic reproductive number.

Part 2: Practice

1. A classroom activity
2. Questions
3. Model development
4. Simulation
5. Parameterization
6. Comparisons
7. Parameter studies
8. Reporting results

A classroom activity

- Students have status cards: green for susceptible, yellow for asymptomatic infectious, red for symptomatic infectious, blue for recovered.
- Students are paired randomly in each time step. If green meets yellow or red, transmission occurs with probability $5/6$.
- Transitions from yellow to red and red to blue take exactly one day.
- Students record, graph, and discuss results.
- Variants can include isolation of symptomatics and vaccination.

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  - Mask usage
  - Initial immunity percentage
- What outputs do we want to measure?
  - Graphs of class populations
  - Maximum size of class I
  - Time for peak of infections
  - Final percentage susceptible
  - Hospitalizations
  - Deaths

Model development - assumptions

We focus on the SEIR epidemic model as an example.

- Four classes:
  - S: Susceptible (can be infected)
  - E: Exposed/Latent (infected, but cannot transmit)
  - I: Infectious (can transmit)
  - R: Removed (recovered or deceased)

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- Two scenario parameters:
  - Add a contact factor δ that accounts for masks.
  - Add an initial immunity fraction V for follow-up scenarios.

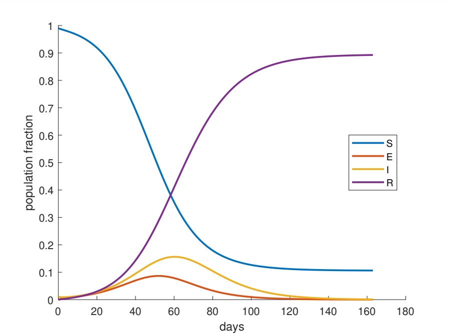
Model development - equations

- Susceptible (S) to latent (E) by transmission at rate $\delta\beta SI$, where δ is a "contact factor" that represents policy and behavior.
- Latent (E) to infectious (I) by incubation at rate ηE.
- Infectious (I) to removed (R) by recovery/death at rate γI.

$$
\begin{aligned}
S' &= \delta\beta SI, & S(0) &= 1-E_0-I_0-V;\\
E' &= \delta\beta SI - \eta E, & E(0) &= E_0;\\
I' &= \eta E - \gamma I, & I(0) &= I_0;\\
R' &= \gamma I, & R(0) &= V.
\end{aligned}
$$

Simulation

- I have simulation programs in Matlab, R, and Excel for SIR, SEIR, and SEAIHRD (Covid-19).
- Students enter scenario data and get a graph.



Parameterization (for Covid-19)

- Best estimates of incubation period and infectious period are $t_e = 5$ and $t_i = 10$, or $\eta = 0.2$ and $\gamma = 0.1$.
- For β, assume we know the early phase doubling time t_d.
  - Assume $E' = \lambda E$, $I' = \lambda I$, where $\lambda = \ln 2/t_d$

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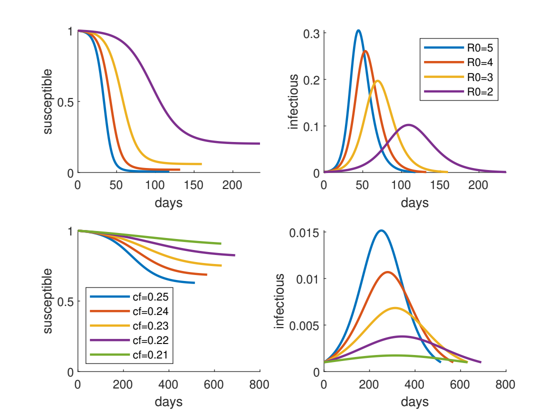
  - Eliminate ρ to get
  $$\beta = \frac{(\lambda+\eta)(\lambda+\gamma)}{\eta}, \tag{1}$$
  - With $t_d \in [3.5, 4.0]$, we get
  $$\mathcal{R}_0 = \beta t_i = \frac{(\lambda+\eta)(\lambda+\gamma)}{\eta\gamma} \in [5.1, 5.9].^* \tag{2}$$

Comparisons

- Vary infectivity β with no interventions or immunity. Examine time series for S and I. (top)
- Vary contact factor (with $R0 = 5.9$ for Covid-19). (bottom)



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  - Use Excel with a template.
    - Students bring some prior experience.
    - Limited programming capacity.
  - Provide Matlab programs (usable with Octave).
    - Some modifications needed.
    - If properly designed, the modifications are minimal!

Making programming easy – SEIR_simplot.m

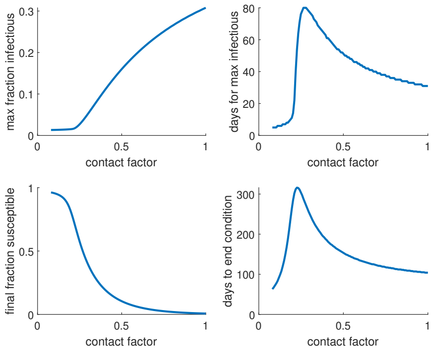
```
%% DEFAULT SCENARIO DATA
beta = 0.5;
etc
V = 0;

%% INDEPENDENT VARIABLE DATA
xvals = [0.5,0.4,0.3,0.2];

%% COMPUTATION
for n=1:N
   beta = xvals(n);
   [S,E,I,R] = seir_sim(beta,eta,gamma,E0,I0,V)
   etc
end
```

Parameter studies

- How does the extent of mask use affect key outcomes: maximum I, final S, times for these events?
  - Use SEIR_paramstudy.m.



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  - Verbal *descriptions*, like "The function increases to a maximum and then decreases to 0", are not enough.
  - Insist on *explanations* and/or *interpretations*, like "The infectious population reaches a maximum because transmission slows down as the number of susceptibles decreases."

Resources

- Thanks for "coming"!
- See **https://www.math.unl.edu/SIR-modeling** for
  - Details on the classroom activity, including directions for an online implementation;
  - Materials for using spreadsheets to teach epidemic modeling;
  - Links to some useful resources.
- See **https://www.math.unl.edu/covid-module** for similar spreadsheet materials.
- These pages will be updated soon to include Matlab materials, SEIR versions, and notes and videos from my epidemiology class just finished.
- **Shoot me an email to receive updates on my materials.**