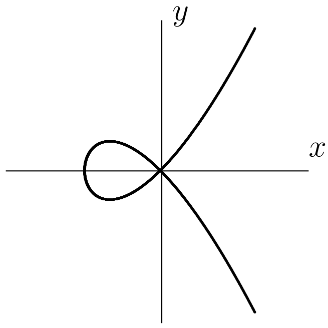
M953, Homework 1, due Friday, January 18, 2013

Instructions: Do any three problems.

(1) Let $n > 0$ and $d \geq 0$ be integers. Define the set of configurations of d cows and n fences to be the set of all strings of letters using only the letters l and o, where l occurs exactly d times and o occurs exactly n times (think of the o's as being cows and the l's as being fences). Let A be the set of monomials of degree at most d in n variables. Let B be the set of monomials of degree exactly d in $n+1$ variables. Let C be the set of configurations of d cows and n fences.
   (a) Show that there is a bijection $A \to B$.
   (b) Show that there is a bijection $B \to C$.
   (c) Show that C has $\binom{d+n}{n}$ elements.

(2) Find a polynomial parametrization of $y^2 - x^2(x+1) = 0$. (Such a parametrization was used to draw the graph at right.)



Graph of $y^2 = x^3 + x^2$.

(3) Do Problem 1.2 on p. 8 of the text.

(4) Let k be a field and let $f(y_1, \ldots, y_n) \in k[y_1, \ldots, y_n]$. Note that $f(y_1, \ldots, y_n) - f(z_1, \ldots, z_n) \in k[y_1, \ldots, y_n, z_1, \ldots, z_n]$. Show that $f(y_1, \ldots, y_n) - f(z_1, \ldots, z_n)$ is an element of the ideal $(y_1 - z_1, \ldots, y_n - z_n)$.

(5) Let k be a field. Let $f : k[y_1, \ldots, y_n] \to k[x_1, \ldots, x_m]$ be a k-homomorphism (i.e., a ring homomorphism such that $f(c) = c$ for all $c \in k$). Let $g_i = f(y_i) \in k[x_1, \ldots, x_m]$, $i = 1, \ldots, n$ and let $I \subseteq k[x_1, \ldots, x_m, y_1, \ldots, y_n]$ be the ideal $(y_1 - g_1, \ldots, y_n - g_n)$. Prove that $\ker(f) = I \cap k[y_1, \ldots, y_n]$.