Homework 7, due Thursday, November 1, 2012

Do any 4 of the 5 problems. Each problem is worth 25 points. Solutions will be graded for correctness, clarity and style.

(1) Let $\Gamma \subset \mathbf{R}^2$ be the graph of a function $f : \mathbf{R} \to \mathbf{R}$. If $\mathbf{R}$ and $\mathbf{R}^2$ are given the standard topologies (and as we now know, the standard topology on $\mathbf{R}^2$ is the same as the product topology) and if f is continuous with respect to the standard topologies, show that Γ is closed. [Hint: Define $F : \mathbf{R}^2 \to \mathbf{R}$ by $F((a, b)) = f(a) - b$. Show that F is continuous and that $\Gamma = F^{-1}(\{0\})$. You may assume that $+ : \mathbf{R}^2 \to \mathbf{R}$, defined as $+((a, b)) \mapsto a + b$, is continuous, and that $- : \mathbf{R} \to \mathbf{R}$, defined as $-(a) = -a$, is continuous.]

(2) Let $\pi_2 : \mathbf{R}^2 \to \mathbf{R}$ be defined by $\pi_2((x, y)) = y$. If $\mathbf{R}$ and $\mathbf{R}^2$ are given the standard topologies, show that π_2 is not a closed map (i.e., give an example of a closed subset $C \subseteq \mathbf{R}^2$ such that $\pi_2(C)$ is not closed).

(3) Give an example of a relation R on a set S which is reflexive and symmetric but not transitive.

(4) Give an example of a relation R on a set S which is reflexive and transitive but not symmetric.

(5) Give an example of a relation R on a set S which is transitive and symmetric but not reflexive.