Practice Quiz 5 Solutions

[1] (a) Problem 15, p. 712: Just sketch two bell-shaped curves which peak at the same value (so μ is the same) but which have different widths, like data sets I and II just before problem 13. The one which is wider has the larger standard deviation σ.

(b) Problem 16, p. 712: Just sketch a bell-shaped curve, and sketch another identical copy of it but slid over a bit so that the two have peaks at different spots. They have different means μ since the peaks are at different spots, but otherwise they are the same (in particular they have the same widths) so they have the same standard deviations σ.

[2] (a) An ACT score of 29 is 9 points above the mean. Since σ is 6, this is 1.5σ. Since z is just the number times σ that the score is above or below the mean, and since 29 is 1.5σ above the mean, we see $z = 1.5$. Note: when the score is below the mean, z is negative. So an ACT of 17 gives $z = -0.5$. (b) The proportion of test takers between the mean of 20 and 1.5σ above the mean is 43.32% according to the table. Since 50% are above the mean, there are $50 - 43.32 = 6.68\%$ above 1.5σ. In other words, 6.68% receive scores of 29 or better. (c) Note that to have 69.15% of test takers receive better scores than you, you're in the bottom half, so you must have gotten a score below the mean. Thus the percentage of test takers that got scores between your score and the mean of 20 is $69.15 - 50 = 19.15\%$. The table value for 0.1915 is $z = 0.5$; i.e., $(1/2)\sigma$ below the mean, or $20 - (1/2)6 = 20 - 3 = 17$. So you must have gotten a score of 17.

[3] Since $n = 2104$, and $\hat{p} = 0.45$, we get $\hat{s} = \sqrt{(.45)(.55)/2104} = 0.0108$. A 95% confidence interval is just the interval ranging from $2\hat{s}$ below $\hat{p}$ to $2\hat{s}$ above $\hat{p}$. I.e., the confidence interval is $0.45 \pm .0216$ or 42.84% to 47.16%.

[4] (a) We must find the sample size n such that we get a value for $\hat{s}$ which gives us the desired accuracy of 1%; i.e., such that $2\hat{s} = 0.01$. But $\hat{s} = \sqrt{(0.02)(0.98)/n}$, so we need $2\sqrt{(0.02)(0.98)/n} = 0.01$. We just need to solve this for n, either by guess and check or by algebra. Either way we get $n = 784$.

(b) Assuming a 95% confidence interval, the margin of error of 4% means that the standard error is half that, or 2%, so $\hat{s} = 0.02$. But using the formula $\hat{s} = \sqrt{\hat{p}(1-\hat{p})/n}$ and given that $\hat{p} = 39\%$, we get $0.02 = \sqrt{0.39(0.61)/n}$ which we solve to find $0.0004 = 0.39(0.61)/n$ or $n = 0.39(0.61)/0.0004 = 594.75$ which rounds to 595.