[1] An election is run. The candidates are Paul (P), Tom (T), Sally (S), and Ann (A). There are 17 voters. Here is a tabulation of their preference lists:

# Voters	5	4	4	2	2
First place	S	P	A	T	A
Second place	P	T	S	P	P
Third place	A	S	T	A	S
Fourth place	T	A	P	S	T

(a) Determine the vote totals using plurality voting. Who is the winner?
Just count the number of first place votes for each candidate: S: 5 P: 4 A: 6 T: 2
Ann has the most votes so she wins.
(b) Who wins if Sally drops out of the race?
If Sally drops out that means that Paul gets her first place votes now (look at the first preference list).
The vote totals are now: P: 9 A: 6 T: 2 Thus Paul now wins.
(c) Do (a) and (b) give an example of a violation of a fairness criterion? If so which one? Explain.
The irrelevant alternatives criterion is violated: Sally (a loser) swung the election from Ann to Paul.
(d) Determine the vote totals using the Borda count. Who is the winner?
P: 47 A: 42 S: 46 T: 35 Paul wins.
(e) Does (d) give an example of a violation of a fairness criterion? If so which one? Explain.
No; although Paul wins the Borda count, while Ann won the plurality, this is not a violation of
the majority Criterion, since Ann did not have a majority.
(f) Indicate the order of elimination using plurality with elimination voting. Who wins?
T goes first, then S, then A leaving P the winner.
(g) Suppose we switch P and A in the last column. Who now wins using plurality with elimination voting?
Again T goes first, but then A, and then P, leaving S the winner.
(h) Does (g) give an example of a violation of a fairness criterion? If so which one? Explain.
The monotonicity criterion is violated, since the winner before moved up, but that made him lose.
(i) Determine the vote totals using pairwise comparison voting. Who is the winner?
P: 2 S: 3 A: 1 T: 0 S is the winner.
[2] Consider the weighted voting system [20 | 13, 8, 7, 4].
(a) Which if any of the voters are dummies? Explain.
The voter of weight 4 is a dummy since no coalition ever needs those 4 votes
to reach the quota; each coalition is either already at the quota or the 4 votes aren't enough.
(b) Which if any of the voters have veto power? Explain.
To have veto power the sum of the rest of the votes must be less than the quota.
Thus the voter with weight 13 is the only one with veto power.
(c) Which if any of the voters are dictators? Explain.
There are no dictators. A dictator weight must be as big as or bigger than the quota.
(d) What is the Banzhaf power index of each voter? Let A, B, C and D be the players, where A has 13 votes, B has 8, etc.
There are six winning coalitions. Here is each one followed by which players are critical:
{A,B,C,D}: A; {A,B,C}: A; {A,B,D}: A, B; {A,C,D}: A, C; {A,B}: A, B; {A,C}: A, C.
Thus A is critical 6 times, B and C 2 times each, and D never, so 6+2+2 = 10.
Thus the Banzhaf indices are: 6/10 for A, 2/10 for B and C, and 0/10 for D.
[3] A local charity is giving away a 12 pack of root beer for a donation of $20. Bobby, Mikey and Janey pool their money to make a $20 donation. Bobby contributes $8, Janey $7, and Mikey $5.
(a) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Hamilton's method is used to deal with fractional parts of cans? Fill in the table below to show your answer.

Child	Standard Quota	Hamiltonian Apportionment
Bobby	12(8/20) = 4.8	5
Janey	12(7/20) = 4.2	4
Mikey	12(5/20) = 3	3

(b) How should the cans be apportioned between them if each child's apportionment is proportional to that child's contribution, assuming that Jefferson's method is used to deal with fractional parts of cans? Fill in the table below to show your answer, and indicate the standard divisor D and the modified divisor d which you are using.

Standard divisor D = ___20/12 = 5/3 = 1.667____
Note the roundings down of the standard quotas add up only to 11 so we need to reduce the standard divisor to get our modified divisor. I'll try d = 1.5 as a first guess for the modified divisor d. That turns out to work.

Child	Standard Quota	Modified Quota	Jeffersonian Apportionment
Bobby	4.8	8/1.5 = 5.333	5
Janey	4.2	7/1.5 = 4.667	4
Mikey	3	5/1.5 = 3.333	3

[4] A school district assigned 35 instructional assistants to five schools based on enrollment figures. The budget allowed for the hiring of 2 additional assistants. Consider the following apportionment numbers before and after the increase in instructional assistants. Is this an example of a paradox? If so, which paradox has occurred? Explain.

School	Original Apportionment	New Apportionment
Cascades	9	9
Seven Oaks	11	10
Riverview	6	7
Pioneer	4	5
Hamilton Creek	5	6

This is an example of an increase in the total number of items to be apportioned causing a reduction in someone's apportionment; i.e., it is an example of the Alabama paradox.