M104, Solutions to Practice Quiz for Quiz 9, April 10, covering sections 7.1 and 7.2

This practice quiz does not count for points, but I will use some of these problems as models which I'll follow for the real Quiz 9 on Wednesday, April 10, 2013.

(1) Find the following indefinite integrals:

(a) $\int \Big(x^2 - \dfrac{1}{x^4}\Big)dx$ (b) $\int e^x dx$ (c) $\int \dfrac{3x^5 + x^3}{x^4} dx$

(a) $\int \Big(x^2 - \dfrac{1}{x^4}\Big)dx = \int \big(x^2 - x^{-4}\big)dx = \dfrac{x^3}{3} - \dfrac{x^{-3}}{-3} + C = \dfrac{x^3}{3} + \dfrac{x^{-3}}{3} + C$

(b) $\int e^x dx = e^x + C$

(c) $\int \dfrac{3x^5 + x^3}{x^4} dx = \int (\dfrac{3x^5}{x^4} + \dfrac{x^3}{x^4})dx = \int (3x + x^{-1})dx = \dfrac{3}{2}x^2 + \ln|x| + C$

(2) Find $C(5)$ if $C'(x) = 3x^2 - 2x$ and $C(2) = 6$.

First, $C(x) = \int C'(x)dx = \int(3x^2 - 2x)dx = x^3 - x^2 + C$.
So $6 = C(2) = 2^3 - 2^2 + C = 8 - 4 + C = 4 + C$, hence $C = 2$.
Thus $C(x) = x^3 - x^2 + 2$, so $C(5) = 5^3 - 5^2 + 2 = 125 - 25 + 2 = 102$.

(3) Find the following indefinite integrals:

(a) $\int xe^{x^2} dx$ (b) $\int (6x + 2)^{15} dx$

(c) $\int 6(x^3 + x)^5(3x^2 + 1)dx$ (d) $\int f'(g(x))g'(x)dx$

(a) Let $u = x^2$. Then $du = 2xdx$ so $dx = \frac{du}{2x}$. Hence $\int xe^{x^2}dx = \int xe^u \frac{du}{2x} = \int \frac{1}{2}e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$.

(b) Let $u = 6x + 2$. Then $du = 6dx$ or $dx = \frac{du}{6}$, so $\int(6x + 2)^{15}dx = \int u^{15}\frac{du}{6} = \frac{u^{16}}{16}\frac{1}{6} + C = \frac{u^{16}}{96} + C = \frac{(6x+2)^{16}}{96} + C$.

(c) Let $u = x^3 + x$, so $du = (3x^2 + 1)dx$ or $dx = \frac{du}{3x^2+1}$. Hence $\int 6(x^3 + x)^5(3x^2 + 1)dx = \int 6u^5(3x^2 + 1)\frac{du}{3x^2+1} = \int 6u^5 du = u^6 + C = (x^3 + x)^6 + C$.

(d) Let $u = g(x)$, so $du = g'(x)dx$ or $dx = \frac{du}{g'(x)}$. Thus $\int f'(g(x))g'(x)dx = \int f'(u)g'(x)\frac{du}{g'(x)} = \int f'(u)du = f(u)+C = f(g(x))+C$. Alternatively, $(f(g(x)))' = f'(g(x))g'(x)$ so $\int f'(g(x))g'(x)dx = \int(f(g(x)))'dx = f(g(x)) + C$.