M104, Solutions for Practice Quiz 8, covering section 6.6

(1) Use differentials to estimate 9^3. (I.e., use the line tangent to the function $f(x) = x^3$ at the point $x = 10$.)

Answer: The formula is that $f(x + \Delta x) \approx f'(x)(\Delta x) + f(x)$, so in our case we have $9^3 = f(9) = f(10 - 1) \approx f'(10)(-1) + f(10)$. Here $f(x) = x^3$, so $f(10) = 1000$, and $f'(x) = 3x^2$ so $f'(10) = 300$. Thus we get $9^3 \approx f'(10)(-1) + f(10) = -300 + 1000 = 700$. (The exact answer is $9^3 = 729$, so our estimate is correct to within about 4%.)

(2) Use differentials to estimate 20.5^2. (Decide for yourself what function $f(x)$ to find the tangent line for and at what value of x to find the tangent line, but explicitly state what $f(x)$ you're using and at what value of x, and what the value of Δx is.)

Answer: The function is $f(x) = x^2$. The point of tangency is $x = 20$. The formula is $f(x+\Delta x) \approx f'(x)(\Delta x)+f(x)$, where $f'(x) = 2x$, and we have: $f(20) = 20^2 = 400$, $f'(20) = 2(20) = 40$ and $\Delta x = 0.5$, so $20.5^2 = f(20 + 0.5) \approx f'(20)(0.5) + f(20) = 40(0.5) + 400 = 420$. (The exact answer is $20.5^2 = 20 * 21 + 0.25 = 420.25$, so our estimate is correct to within about 0.05%; i.e., way less than 1%.)

(3) Let q be the quantity of some product that can be sold at a unit price p. When p is \$5, assume that $q = 100,000$ and that $\frac{dq}{dp} = 500$.

(a) Find the revenue when $p = 5$.

Answer: The revenue as a function of p is $R(p) = pq$. Thus $R(5) = 5 * 100000 = 500,000$.

(b) Use differentials to estimate how much the revenue increases if p is increased by 10 cents.

Answer: The function is $R = pq$, so $\frac{dR}{dp} = q + p\frac{dq}{dp}$. Thus at $p = 5$, $q = 100,000$ and $\frac{dq}{dp}(5) = 500$, we have $\frac{dR}{dp}(5) = 100,000 + 5 * 500 = 102,500$. The formula for the approximation to R at $p + \Delta p$ is $R(p + \Delta p) \approx \frac{dR}{dp}(p)(\Delta p) + R(p)$, so $R(5.10) = R(5 + 0.10) \approx \frac{dR}{dp}(5)(0.10) + R(5) = 102500(0.10) + 500000 = 510250$. Thus the revenue increased \$10,250.