M104, Practice Quiz 5

This practice quiz does not count for points, but I will use some of these problems as models which I'll follow for the real Quiz 5 on Wednesday, February 20, 2013.

(1) Determine which of the following investments is best and which is worst. Explain how you obtained your answer.
   (a) Assume continuous compounding at an interest rate $r = 8\%$.
   (b) Assume semiannual compounding at an interest rate $r = 8.1\%$.
   (c) Assume a doubling time of 8.65 years.

   Answer: We can either find the doubling times, the effective rates or the growth of a dollar over a year. For (a) a dollar will grow to $e^{.08} = 1.083287$ (so the effective rate is $1.083287 - 1 = 8.3287\%$). For (b) a dollar will grow to $(1 + \frac{.081}{2})^2 = 1.08264$ (so the effective rate is $1.08264 - 1 = 8.264\%$). For (c), P dollars will grow to $A = P2^{t/d}$ after t years if the doubling time is d, so 1 dollar over a year with a doubling time $d = 8.65$ years will grow to $2^{1/8.65} = 1.08343$ (so the effective rate is $1.08343 - 1 = 8.343\%$). Thus (c) is best and (b) is worst. You'd get the same answer using doubling times. For (a) the doubling time is $\ln(2)/.08 = 8.664$ years, for (b) the doubling time is $\ln(2)/ln(1.08264) = 8.7295$ years, and for (c) we are given that it is 8.65 years. So (c) is best since it has the shortest doubling time, and (b) is worst since it has the longest doubling time.

(2) Find $\dfrac{dy}{dx}$ for $y = e^{x^2}$, and find $\dfrac{dy}{dx}$ for $y = (e^x)^2$.

   Answer: We have
$$(e^{x^2})' = e^{x^2}(x^2)' = e^{x^2}2x$$
   but
$$((e^x)^2)' = 2(e^x)(e^x)' = 2(e^x)(e^x) = 2e^{2x}.$$
   You might have done the second one a different way, since $(e^x)^2 = e^{2x}$ so
$$((e^x)^2)' = (e^{2x})' = (e^{2x})(2x)' = 2e^{2x}.$$

(3) Find $\dfrac{dy}{dx}$ for $y = \ln(x^2)$, and find $\dfrac{dy}{dx}$ for $y = (\ln(x))^2$.

   Answer: We have
$$(\ln(x^2))' = \frac{1}{x^2}(x^2)' = \frac{1}{x^2}2x = \frac{2}{x}$$
   but
$$((\ln(x))^2)' = 2(\ln(x))(\ln(x))' = 2(\ln(x))\frac{1}{x} = \frac{2\ln(x)}{x}.$$
   You might have done the first one a different way, since $\ln(x^2) = 2\ln(x)$ so
$$(\ln(x^2))' = (2\ln(x))' = 2(\ln(x))' = \frac{2}{x}.$$

(4) According to the UN, world population (in millions) t years after 1960 is approximately given by $P = 3100e^{0.0166t}$. Find the rate of growth in 1960 (in millions of people per year) and find the rate of growth now. Also, find the doubling time.

   Answer: The doubling time is $\dfrac{\ln 2}{0.0166} = 41.76$ years. Also, $\frac{dP}{dt} = 3100(e^{0.0166t})' = 3100e^{0.0166t}(0.0166t)' = 3100e^{0.0166t}0.0166 = 3100(0.0166)3100e^{0.0166t} = 51.46e^{0.0166t}$, so in 1960 (i.e., when $t = 0$) the rate of increase was $P'(0) = 51.46e^{0.0166*0} = 51.46$ million people per year. In 2013 (i.e., when $t = 53$), it is $P'(53) = 51.46e^{0.0166*53} = 124$ million people per year. This is equivalent to adding more than the whole population of the USA to the world every 3 years.