Lecture Notes on Predation Functional Forms

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Holling's original derivation:

X – number of prey, $Y = 1$ – number of predator

T – total time

h – handling time per prey

a – encounter rate or discovery rate

X_c – number of captured prey (in T) by the single predator

Then the time left for searching

$$T_s = T - h \cdot X_c$$

and Holling's equation follows

$$X_c = T_s \cdot a \cdot X = (T - h \cdot X_c) \cdot a \cdot X$$

Solve it to get the so-called Holling's Type II functional form, or disc function:

$$X_c = \frac{TaX}{1 + haX} \qquad \Leftrightarrow \qquad \frac{X_c}{T} = \frac{aX}{1 + haX}$$

with the latter being the per-predator predation rate.

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Generalize Holling's equation to $Y \geq 1$ many predators to get

$$X_c = T_s aX = (T - hX_c)a(X - X_c(Y-1))_+ = (T - hX_c)aX\left(1 - \frac{X_c(Y-1)}{X}\right)_+$$

where $x_+ = x$ if $x \geq 0$ and $x_+ = 0$ if $x \leq 0$. Denote by $\epsilon = \frac{X_c(Y-1)}{X}$, and substitute $X(1-\epsilon)$ for X in Holling's disc function to get

$$X_c = \frac{TaX(1-\epsilon)_+}{1 + haX(1-\epsilon)_+} := f(\epsilon).$$

This is equivalent to a quadratic equation in X_c and X_c can be solved exactly. However, it is a complicated form, but can be simplified under one condition that $\epsilon \ll 1$. Use Taylor's approximation to get $X_c = f(\epsilon) \approx f(0) + f'(0)\epsilon$ which is a linear equation in X_c. Solve it to have

$$X_c = \frac{TaX}{1 + haX + \frac{Ta(Y-1)}{1+haX}} = \frac{TaX(1+haX)}{(1+haX)^2 + Ta(Y-1)}.$$

The per-predator predation rate becomes

$$\frac{X_c}{T} = \frac{aX(1+haX)}{(1+haX)^2 + Ta(Y-1)}.$$

We note that $Y = 1$ reduces it to Holling's Type II form, and replacing the factor $\frac{Ta}{1+haX}$ of $Y - 1$ by Beddington's constant interference factor bt_w gives Beddington's form. Here b is Beddington's predator's interference encounter rate and t_w is the wasted interfering time per predator per encounter. Our result shows that such interference decreases with the prey density, which seems more realistic.

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Online Supplemental Materials in Matlab: http://www.math.unl.edu/ bdeng1/Research/biomath/BioDiversity/