Take Home Final Exam MATH 939, Due Dec 20

1. Construct a map f from $I = [0, 1]$ to $\mathbb{R}$ so that the set $C = \{x : f^n(x) \in I,\ n \geq 0\}$ is a Cantor set of positive Lebesgue measure.

2. Let f and $H_{s_0 s_1 \cdots s_n \cdots}$ be defined as in Lecture Notes 4. Modify the proof of Theorem 1 to show $H_{s_0 s_1 \cdots s_n \cdots}$ is the graph of of a function $y = \theta(x)$.

3. Let f be a circle map defined by $f(x) = (x + c)\ (\mathrm{mod} 1)$ for some constant c. Prove

a. Every orbit is periodic if c is rational.

b. Every orbit is dense if c is irrational.

4. Let $\mathcal{R} : X_0 \to X_0$ be the spike-renormalization defined in class. Prove that it has the property of sensitive dependence on initial conditions.