Course Title: Introduction to Chaos Theory.
Course Number: Math939
Semester: Fall, 2019
Time: 12:30-1:45 TR
Room: Avery 11

**Description:** General Relativity and Quantum Mechanics are two paradigm shifts in science for the last century. Chaos Theory is regarded by many as the third paradigm shift because it has changed the centuries-held view that deterministic dynamical systems are predictable. This course is an introduction to this theory.

Topically, we will limit our studies to iterative maps in metric spaces from $R^1$ to $R^n$ and then to a $L^1$ functional space. Math825-826 is the only prerequisite for the course. We will develop all new and necessary tools throughout the course as needed.

We will study many basic issues about the dynamics of iterative maps. Among them include existence and stability of fixed and periodic orbits, bifurcations of periodic orbits, various working definitions of chaotic orbits in terms of sensitive dependence on initial conditions, probability distribution of orbits, and Lyapunov exponents of orbits. We will also study topological conjugates of chaotic maps to symbolic dynamics, as well as geometric structures of the phase spaces in terms of manifolds and foliations which are invariant to the maps.

Here are some of the theorems we will study for the course: the Perron-Frobenius Theorem for transition probability matrixes on Markov chains, the Center Manifold Theorem for fixed points of maps, the Universal Number Theorem for a renormalization operator on 1-d maps which modeling spike generation for neurons that says the first integer 1 is a universal number for the neural maps, and the Big Chaos Theorem which says that every dynamical system can be topologically embedded into a chaotic set of the renormalization operator in a $L^1$-subspace of the unit interval.