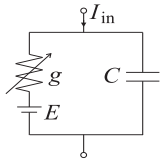
Homework

1. Approximate the solution to the initial value problem

$$\begin{cases} x' = f(t,x) = -2x + te^{-t} \\ x(1) = 3. \end{cases}$$

in the interval $[1, 1.2]$ in 5 discrete points including the initial point. Sketch the solution. Do not use computer programs.

2. Consider the RC-circuit shown. Instead of piecewise linear functions, assume the nonlinear VI-characteristics is given by



$$V = f(I) = 3(I^3 - 2I^2 + I).$$

Assume the following parameter values for the circuit

$$C = 0.1,\ E = 0.2.$$

For numerical approximation and simulation, convert the circuit equations to the following system of differential equations

$$\begin{cases} V_C' = -(I_g + I_{\text{in}})/C \\ I_g' = F(V_C + E, I_g)/\epsilon \end{cases}$$

where $F(V,I) = V - f(I) = V - 3(I^3 - 2I^2 + I)$ and use $\epsilon = 0.001$. Modify appropriate `m.files` from the LectureNotes folder `matlab/mfilechcircuits/Scircuit` to do the following.

   1. For $I_{\text{in}} = -0.6$, sketch the graphs of both V_C-nullcline and I_g-nullcline in the $V_C I_g$ phase plane.
   2. For $I_{\text{in}} = -0.6$, use the `Matlab` solver `ode15s` to approximate the solution starting at the resting potential point $(-E, 0)$ for a time duration from $t = 0$ to $t = T = 1$. Plot both time series of the solution curves over the time interval $[0, T]$.
   3. Plot the solution curve (i.e. parameterized curve $(V_C(t), I_g(t))$ from (2) in the same phase plane of (1).
   4. Repeat the same tasks for parameter value $I_{\text{in}} = -0.1$. Comment on differences between the two cases.
   5. $I_{\text{in},0}$ is an oscillation **threshold** if the circuit dynamics is oscillatory for parameter values of I_{in} immediately on one side of $I_{\text{in},0}$ but not the other side. How many thresholds for this circuit and what are they?

6. A square-burster circuit was constructed in the LectureNotes folder `matlab/mfilechcircuits/SquareBursterCircuit/PiecewiseLinearIV`. Use the mfile `SBsimulation1.m` to find an L value so that the circuit produces bursts of 6 spikes. Submit all figures from the simulation.