M428 Final Exam

Name: _____ 8-Digits NUID: _____

Instructions:

(1) Use ballpoint pens, or ink pens to write if you work on paper before photographing your solutions for submission. Do not use pencils.

(2) Zoom audio and video must be on all the time. No headphones. You must be alone in a room. No simulated background on your video feed. Position your webcam in such a way that I can see your writing area.

(3) All messaging systems, such as email, texting, facebook, etc. must be turned off.

(4) Exam problems and/or their solutions are not allowed to be shared with anyone or any online site, in part or in whole or in paraphrase.

(5) Textbook, lecture notes and video lectures from Canvas, exercises and homework solutions, calculators, Matlab, Excel Solver, personal notes are allowed. Any other study aids must be requested for clearance, in advance. Any form of cheating will not be tolerated.

(6) Do not copy the problem statement. But clearly label your solutions to problem questions.

(7) Scan and submit your work to Gradescope before the allotted time ends.

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1 (30 pts) Consider the linear programming problem

$$\begin{aligned} \text{Maximize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \le 1 \\ & 3x_1 + x_2 \le 13 \\ & 2x_1 + x_2 = 7 \\ & x_1 \ge 0,\ x_2 \ge 0. \end{aligned}$$

(a) Use the Big-M simplex method to solve the problem. You must show each feasible row echelon form to receive credit. Intermediate forms between feasible row echelon forms can be omitted but substituted by instructions on row operations.

(b) Find the matrix E that transform the initial tableau in matrix to the final tableau in matrix.

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2 (30 pts) Two politicians soon will be starting their campaigns against each other for a political office. Each must now select the main issue(s) she will use throughout the campaign. Each has a few advantageous issues from which to choose, but the relative effectiveness of each one would depend upon the issue chosen by the opponent. In particular, the estimated increase in the vote for politician 1 (expressed as a percentage of the total vote) resulting from each combination of issues is as follows:

		Politician 2			
	Strategy	**1**	**2**	**3**	**4**
	1	1	-3	2	1
Politician 1	**2**	2	3	0	-2
	3	0	4	-1	-2

However, because of the time constraint, each candidate must make her own choice before learning the opponent's choice.

(a) Eliminate dominated strategies as much as possible for each politician.

(b) The politicians will use the remaining strategies throughout the campaign repeatedly. Set up a linear programming problem for the optimal strategy for Politician 1.

(c) Use Excel/Solver or any means to find the optimal strategies for both politicians. (Include screenshots of the methods for supporting work.)

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3 (30 pts) Customers arrive at a queueing system according to a Poisson process with a mean arrival rate of 2 customers per minute. The service time has an exponential distribution with a mean of 1 minute. An unlimited number of servers are available as needed so customers never wait for service to begin.

(a) Construct the rate diagram for this queueing system.

(b) Write down the first three balance equations for the birth-and-death process.

(c) Find the expected waiting time of a customer in the queue W_q.

(d) Find the expected waiting time of a customer in the system W.

(e) Find the expected number of customers L in the system.

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4 (30 pts) Let

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

be the transition matrix for a Markov process.

(a) Construct its Markov chain transition diagram on four states: $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$.

(d) Find the probability, $f_{13}^{(4)} = P\{T_{13} = 4\}$, for the first-passage-time random variables, T_{13}, that takes exact four steps to go from state **1** to **3**.

(c) Identify the absorbing state(s) and find all absorbing probabilities that state **1** is eventually absorbed. (You must show work to receive full credit for all parts).

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5 (30 pts) You are searching on a website for a cheap flight from either Lincoln (K) or Omaha (O) to Tokyo (T). The website is ready for action. Here are the possible routes: Lincoln to Chicago (C) or Denver (D), Omaha to Chicago (C) or Denver (D), and then Chicago to Tokyo directly or by way of either San Francisco (S) or Los Angeles (L), or Denver-San Francisco-Tokyo or Denver-Los Angeles-Tokyo. Let p_{ij} in dollars denote the lowest price between the allowed city i and city j above and they are given in the table below:

	D	C	S	L	T
K	310	310	-	-	-
O	270	150	-	-	-
D	-	-	250	270	-
C	-	-	410	350	1350
S	-	-	-	-	850
L	-	-	-	-	900

(a) Set up a multi-staged bipartite diagram for the problem.

(b) Find the optimal solution by the graphical method.

(c) If a shuttle bus from Lincoln to Omaha airport costs $50, where should you catch a flight, Lincoln or Omaha?

Bonus (15 pts) (This will be used if your total score for problems 1–5 above is below 120 and in such a case your final score will be capped at 120.) Continue on hwk14_w5w #1.

(a) Define the vertex sets V_i for the problem and write the adjacency matrices for the multi-staged bipartite graph.

(b) Find the optimal solution by the algebraic backward algorithm.

End of Final Exam