

M428 Exam 2

Name: _____

8-Digits NUID: _____

Instructions:

- (1) Use ballpoint pens, or ink pens to write if you work on paper before photographing your solutions for submission. Do not use pencils.
- (2) Zoom audio and video must be on all the time. No headphone. You must be alone in a room. No simulated background on your video feed. Position your webcam in such a way that I can see your writing area.
- (3) All messaging systems, such as email, texting, facebook, etc. must be turned off.
- (4) Exam problems and/or their solutions are not allowed to be shared with anyone or any online site, in part or in whole or in paraphrase.
- (5) Textbook, lecture notes and video lectures from Canvas, exercises and homework solutions, calculators, Matlab, Excel Solver, personal notes are allowed. Any other study aids must be requested for clearance, in advance. Any form of cheating will not be tolerated.
- (6) Do not copy the problem statement. But clearly label your solutions to problem questions.
- (7) Scan and submit your work to Gradescope before the allotted time ends.

1 (30 pts) On-line orders for a product come in according to a Poisson process at a mean rate of 30 per hour. Two candidates are applying for the job to fill out the orders (before shipping). Both candidates have an exponential distribution for service time per order, with candidate X having a mean of 1.2 minutes and candidate Y having a mean of 1.5 minutes.

- (a) Find the expected waiting time (in minutes) for an order in the system (before shipping) if X is hired.
- (b) Find the same expected waiting time if Y is hired.
- (c) If the management has decided that for each minute saved for an order the handler/server should make \$1 more, how much more should handler X make per hour than handler Y should?

2 (35 pts) A gas station has 3 gas pumps. The amount of time that a pump works before breaking down has an exponential distribution with a mean of 3 weeks. It has only one mechanics qualified to repair broken pumps, who is able to fix 10 pumps per week and the repair time is also exponentially distributed.

- (a) Let the state of the system be the number of pumps out of work, construct the rate diagram for this queueing system.
- (b) Write down the balance equations.
- (c) What is the fraction of time all pumps are working?
- (d) What is the expected time a broken pump is not in service?

3 (35 pts) Dave's Photography Store has a weekly inventory policy for one brand of camera: if the store has cameras on hand at the end of week, then Dave will do nothing, otherwise Dave will order 2 cameras for the start of next week. Assume the demand for this particular brand of camera has a Poisson distribution for a mean of one camera per week.

- (a) Let D be the demand random variable during each week when the store is in business. Find

$$P(D = 0), P(D = 1), P(D \geq 2).$$

- (b) Let $X(t)$ be the number of cameras in stock at the end of week t . Find the 1-step transition matrix. Choose any two entries of the matrix, say p_{00} , p_{11} , and explain how they are derived.
 - (c) Find the probability that the store has 2 cameras at the end of week 3 given that the store has 1 camera at the end of week 1.
 - (d) Find the expected first passage time from the store's having to order 2 cameras to ordering none.
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