SS2021 M428 Exam 1

Print Your Name Legibly:______________________________ Score:__________

1(30 pts) Consider the linear programming problem

Maximize $z = 3x_1 + 2x_2$
subject to

$$\begin{aligned} x_1 \quad\quad &\le 5 \\ x_1 + x_2 &\le 8 \\ x_1 \ge 0,\ x_2 &\ge 0. \end{aligned}$$

(a) Find the dual LP problem.

(b) Solve the dual LP problem graphically, including all details for credit.

(c) Without solving the primal LP problem, what is its optimal value z^* based on (a) and (b)?

(d) If the right-hand side of the second constraint for the primal LP problem changes from 8 to 7.5, without solving the primal LP problem again what is the primal optimal value z^* expected to be? Explain why.

2(35 pts) The augmented matrix form (tableau form) for a LP problem: maximize $z = c^T x$ subject to $Ax \le b,\ x \ge 0$ is given as follows:

		z	x_1	x_2	x_3	x_4	x_5	rhs
	z	1	-1	-2	-3	0	0	0
basic variable	x_4	0	1	-1	1	1	0	0
	x_5	0	2	1	1	0	1	4

(a) Is $x_1 = x_2 = x_3 = 2$ a feasible point of the problem? Justify your answer.

(b) Find the optimal solution of the problem by using the tableau simplex method only. To receive credit, you must show the feasible echelon form for each iteration of the method together with application of optimality test, ratio test, and elementary row operations, such as 2R1+R0 for example, from one echelon form to the next.

(c) Let $E = E_k \cdots E_1$ where $E_i, i = 1, \ldots, k$, are the elementary matrices corresponding to the elementary row operations used for the simplex method above. What is E?

(d) If the right-hand of the constraint is changed from $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ to $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$, use (c) only to find the new optimal solution. Explain why your solution is the optimal solution. Any other method will not receive the full point.

3(35 pts) For a zero-sum game with the following payoff table

		Player 2		
	Strategy	**1**	**2**	**3**
Player 1	**1**	3	2	-3
	2	-1	1	2

(a) Use the graphical method to find the optimal mixed strategies for Player 1 only.

(b) A fair game is one for which the game value $v = 0$. With everything else the same except for the pay-off for the {2,3}-play

		Player 2		
	Strategy	**1**	**2**	**3**
Player 1	**1**	3	2	-3
	2	-1	1	a

use the graphical method to determine the parameter value a so that the game is a fair game.

(c) Use the graphical method to find the optimal mixed strategies for both players for the fair game (b).

(d) Write down either the primal or the dual linear programming problem for the fair game, and use Excel/Solver to verify the solutions obtained above. Including screenshots for the input data sheet and the sensitivity report sheet from the Solver for supporting work.

The End