17.2-4

a) $L = 0\left(\frac{1}{16}\right) + 1\left(\frac{4}{16}\right) + 2\left(\frac{6}{16}\right) + 3\left(\frac{4}{16}\right) + 4\left(\frac{1}{16}\right) = 2$ which represents the average number of customers in the shop, including those getting their hair cut.

b)

n	# in queue	probability	product
0	0		
1	0		
2	0		
3	1	0.25	0.25
4	2	0.0625	0.125

Lq = 0.375 which represents the average number of customers in the shop waiting to get a haircut.

c) $E(\#\text{ customers being served}) = 1 \cdot P_1 + 2\,(P_2 + P_3 + P_4) =$

$= \frac{4}{16} + 2\left(\frac{6}{16} + \frac{4}{16} + \frac{1}{16}\right) = \frac{13}{8}$

d)

$$W = \frac{L}{\lambda} = \frac{2}{4} = 0.5 = 30 \text{ minutes}$$

$$W_q = \frac{L_q}{\lambda} = \frac{0.375}{4} = 0.094 = 5.625 \text{ minutes}$$

These quantities mean that customers will be in the shop an average of half an hour, including the time to get a haircut, and will have to wait an average of 5.625 minutes before their haircut will begin.

e) W - Wq = 0.5 – 0.094 = 0.406 hours = 24.36 minutes

17.2-3

a) A parking lot is a queueing system for providing parking with cars as the customers, and parking spaces as the servers. The service time is the amount of time a car spends in a space. The queue capacity is 0.

b)

$$L = 0(P_0) + 1(P_1) + 2(P_2) + 3(P_3) = 0(0.2) + 1(0.3) + 2(0.3) + 3(0.2) = 1.5 \text{ cars}$$

$$L_q = 0 \text{ cars}$$

$$W = \left(\frac{L}{\lambda}\right) = \left(\frac{1.5}{2}\right) = 0.75 \text{ hours}$$

$$W_q = \left(\frac{L_q}{\lambda}\right) = \left(\frac{0}{2}\right) = 0 \text{ hours}$$

c) A car spends an average of 45 minutes in a parking space.

17.2-6

The utilization factor ρ represents the fraction of time that the server is busy. The server is busy except when there are zero people in the system. P_0 is the probability of having 0 customers in the system. Hence, $\rho = 1 - P_0$.

17.4-2

$\lambda_n = 2$ for $n \geq 0 \Rightarrow P\{n \text{ arrivals in an hour}\} = \frac{2^n e^{-2}}{n!}$

(a) $P\{0 \text{ arrivals in an hour}\} = e^{-2} = .135$

b) $P\{2 \text{ arrivals in an hour}\} = \frac{2^2 e^{-2}}{2!} = 2e^{-2} = .270$

c) $P\{5 \text{ or more arrivals in an hour}\} = 1 - \sum_{n=0}^{4} P\{n \text{ arrivals in an hour}\}$

$= 1 - e^{-2} - 2e^{-2} - 2e^{-2} - (4/3)\cdot e^{-2} - (2/3)\, e^{-2}$

$= 1 - 7e^{-2} = .0527$

17.4-3

$Pay = 100 \cdot P\{T<2\} + 80 \cdot P\{T>2\} = 100 - 20 \cdot P\{T>2\}$

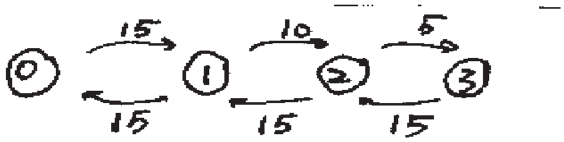
$P\{T_{old} > 2\} = e^{-\frac{1}{4}\cdot 2} = e^{-\frac{1}{2}} = 0.607$

$P\{T_{special} > 2\} = e^{-\frac{1}{2}\cdot 2} = e^{-1}$

$\text{Increase} = Pay_{special} - Pay_{old} = 20\left(P\{T_{old} > 2\} - P\{T_{special} > 2\}\right)$

$= 20\left(e^{-\frac{1}{2}} - e^{-1}\right)$

17.5-5 a)



b) $\begin{cases} 15p_0 = 15p_1 & ① \\ 15p_0 + 15p_2 = 25p_1 & ② \\ 10p_1 + 15p_3 = 20p_2 & ③ \\ 5p_2 = 15p_3 & ④ \end{cases}$

c) $p_0 = p_1$, $p_2 = \frac{5p_1 - 3p_0}{3} = \frac{2}{3}p_0$, $p_3 = \frac{1}{3}p_2 = \frac{2}{9}p_0$

$p_0 + p_1 + p_2 + p_3 = 1 = \frac{26}{9}p_0 \Rightarrow p_0 = \frac{9}{26}$

$p_1 = \frac{9}{26}$ $\quad p_2 = \frac{3}{13}$ $\quad p_3 = \frac{1}{13}$

Or $p_1 = \frac{\lambda_0}{\mu_1}p_0 = p_0$

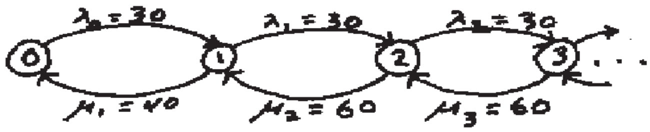
$p_2 = \frac{\lambda_0\lambda_1}{\mu_1\mu_2}p_0 = \frac{2}{3}p_0$

$p_3 = \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}p_0 = \frac{2}{9}p_0$

d) $L = p_1 + 2p_2 + 3p_3 = \frac{27}{26} = 1.04$, $\bar{\lambda} = \lambda_0 p_0 + \lambda_1 p_1 + \lambda_2 p_2 = \frac{255}{26}$
$= 9.81$

$W = L/\bar{\lambda} = \frac{9}{85}$ (hour) $= 0.106$

17.5-9 a)



(b) $P_0 = \left[1 + \sum_{n=1}^{\infty} \frac{\lambda^n}{\mu_1 \mu_2^{n-1}}\right]^{-1} = \left[1 + \frac{\lambda}{\mu_1} \sum_{n=1}^{\infty} \left(\frac{\lambda}{\mu_2}\right)^{n-1}\right]^{-1} = \left[1 + \frac{\lambda}{\mu_1}\left(\frac{1}{1-\frac{\lambda}{\mu_2}}\right)\right]^{-1}$

$= \left[1 + \frac{3}{4}\left(\frac{1}{1-1/2}\right)\right]^{-1} = 2/5 = .4$

$P_n = P_0 \frac{\lambda^n}{\mu_1 \mu_2^{n-1}} = \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)^n$ for $n \geq 1$

c) $L = \sum_{n=0}^{\infty} nP_n = \frac{3}{5}\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n = \frac{3}{5} \cdot \frac{1}{2} \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n-1} = \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{1}{(1-1/2)^2} = \frac{6}{5}$

$L_q = L - (1 - P_0) = 6/5 - (1 - 2/5) = 3/5$

$W = L/\bar{\lambda} = 1/25$

$W_q = L_q/\bar{\lambda} = 1/50$

17.5-11

(a) $L_q = W_q = 0$

(b)

(c) $P_n = \frac{\lambda^n}{n!\,\mu^n} P_0 = \frac{\rho^n}{n!} P_0$

(d) $P_0 = \frac{1}{\sum_{i=0}^{\infty} \frac{\rho^n}{n!}} = \frac{1}{e^{\rho}} = e^{-\rho}$ notice that ρ need not be ≤ 1 in this case.

(e) $W = 1/\mu$, $L = \lambda W = \lambda/\mu = \rho$

17.6-9

a)

$$L = \frac{\lambda}{\mu - \lambda} = \frac{30}{40-30} = 3 \text{ customers}$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{40-30} = 0.1 \text{ hours}$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{30}{40(40-30)} = 0.075 \text{ hours}$$

$$L_q = \lambda W_q = 30(0.075) = 2.25 \text{ customers}$$

$$P_0 = 1 - \rho = 1 - 0.75 = 0.25$$

$$P_1 = (1-\rho)\rho = (1-0.75)0.75 = 0.188$$

$$P_2 = (1-\rho)\rho^2 = (1-0.75)0.75^2 = 0.141$$

There is a 42% chance of having more than 2 customers at the checkout stand.

b)

Data

λ =	30	(mean arrival rate)
μ =	40	(mean service rate)
s =	1	(# servers)

Pr(ω>t) =	3.975E-31
when t =	7

Prob(ω_q>t) =	1.447E-22
when t =	5

P0+P1+P2= 0.578125

Results

L =	3
L_q =	2.25
W =	0.1
W_q =	0.075
ρ =	0.75
P_0 =	0.25
P_1 =	0.1875
P_2 =	0.140625

c)

$$L = \frac{\lambda}{\mu - \lambda} = \frac{30}{60 - 30} = 1 \text{ customer}$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{60 - 30} = 0.033 \text{ hours}$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{30}{60(60 - 30)} = 0.017 \text{ hours}$$

$$L_q = \lambda W_q = 30(0.075) = 0.5 \text{ customer}$$

$$P_0 = 1 - \rho = 1 - 0.5 = 0.5$$

$$P_1 = (1 - \rho)\rho = (1 - 0.5)0.5 = 0.25$$

$$P_2 = (1 - \rho)\rho^2 = (1 - 0.5)0.5^2 = 0.125$$

There is a 12.5% chance of having more than 2 customers at the checkout stand.

d)

Data

λ =	30	(mean arrival rate)
μ =	60	(mean service rate)
s =	1	(# servers)

Pr(ω>t) =	6.283E-92
when t =	7

Prob(ω_q>t) =	3.588E-66
when t =	5

P0+P1+P2= 0.875

Results

L =	1
L_q =	0.5
W =	0.03333333
W_q =	0.01666667
ρ =	0.5
P_0 =	0.5
P_1 =	0.25
P_2 =	0.125

e) The manager should adopt the new approach of adding another person to bag the groceries.

17.6-11

a)

Data

$\lambda =$	2	(mean arrival rate)
$\mu =$	1	(mean service rate)
s =	4	(# servers)

$\Pr(\omega>t)$ =	0.0079018
when t =	5

$\text{Prob}(\omega_q>t)$ =	7.896E-06
when t =	5

P0+P1+P2+P3+P4+P5+P6+P7+P8+P9=

0.997282609

Results

L =	2.17391304
L_q =	0.17391304
W =	1.08695652
W_q =	0.08695652
ρ =	0.5
P_0 =	0.13043478
P_1 =	0.26086957
P_2 =	0.26086957
P_3 =	0.17391304
P_4 =	0.08695652
P_5 =	0.04347826
P_6 =	0.02173913
P_7 =	0.01086957
P_8 =	0.00543478
P_9 =	0.00271739

All the guidelines are currently being met.

b)

Data

$\lambda =$	3	(mean arrival rate)
$\mu =$	1	(mean service rate)
s =	4	(# servers)

$\Pr(\omega>t)$ =	0.0239006
when t =	5

$\text{Prob}(\omega_q>t)$ =	0.0034325
when t =	5

P0+P1+P2+P3+P4+P5+P6+P7+P8+P9=

0.9093317

Results

L =	4.52830189
L_q =	1.52830189
W =	1.50943396
W_q =	0.50943396
ρ =	0.75
P_0 =	0.03773585
P_1 =	0.11320755
P_2 =	0.16981132
P_3 =	0.16981132
P_4 =	0.12735849
P_5 =	0.09551887
P_6 =	0.07163915
P_7 =	0.05372936
P_8 =	0.04029702
P_9 =	0.03022277

The first two guidelines will not be satisfied in one year but the third will be.

c) Five tellers will be needed in a year, 17.6-26 (a) The M/M/s Model:

λ = 6

μ = 4

s = 3

L = 1.737

L_q = 0.237

W = 0.289

W_q = 0.039

P(W > t) = 0.026, where t = 1

P(W_q > t) = 0.237, where t = 0

P_0 = 0.21053
P_1 = 0.31579
P_2 = 0.23684
P_3 = 0.11842
P_4 = 0.05921
P_5 = 0.02961
P_6 = 0.0148
P_7 = 0.0074
P_8 = 0.0037
P_9 = 0.00185
P_{10} = 0.00093
P_{11} = 0.00046
P_{12} = 0.00023
P_{13} = 0.00012
P_{14} = 0.00006
P_{15} = 0.00003
P_{16} = 0.00001
P_{17} = 0.00001
P_{18} = 0
P_{19} = 0
P_{20} = 0

(b) $P\{$a phone is answered immediately$\} = 1 - P\{W_q > 0\} = 0.763$

or, $= P\{$at least one server is free$\} = P_0 + P_1 + P_2$

$= 0.21053 + 0.31579 + 0.23684 = 0.763$

(c) $P\{$n calls on hold$\} = P'_n = P_{n+3} \quad (n \geq 1)$

$P'_0 = P_0 + P_1 + P_2 + P_3 = 0.88158$

(d) The printed measures are in the next page.

$P\{$arriving call is lost$\}$

$= P\{$all three servers are busy$\}$

$= P_3 = 0.13433$

Finite Queue Variation of the M/M/s Model:

λ = 6

μ = 4

s = 3

K = 3

L = 1.299

L_q = 0

W = 0.25

W_q = 0

P_0 = 0.23881
P_1 = 0.35821
P_2 = 0.26866
P_3 = 0.13433
P_4 = 0
P_5 = 0
P_6 = 0
P_7 = 0
P_8 = 0
P_9 = 0
P_{10} = 0
P_{11} = 0
P_{12} = 0
P_{13} = 0
P_{14} = 0
P_{15} = 0
P_{16} = 0
P_{17} = 0
P_{18} = 0
P_{19} = 0
P_{20} = 0

17.6-27 (a) This is a finite Calling Population of M/M/s Model.
Here $\lambda = 1$, $\mu = 2$, $S = 1$, $N = 3$

(b) Finite Calling Population of the M/M/s Model:

$\lambda = 1$

$\mu = 2$

$s = 1$

$N = 3$

$L = 1.421$

$L_q = 0.632$

$W = 0.9$

$W_q = 0.4$

$P_0 = 0.21053$
$P_1 = 0.31579$
$P_2 = 0.31579$
$P_3 = 0.15789$
$P_4 = 0$
$P_5 = 0$
$P_6 = 0$
$P_7 = 0$
$P_8 = 0$
$P_9 = 0$
$P_{10} = 0$
$P_{11} = 0$
$P_{12} = 0$
$P_{13} = 0$
$P_{14} = 0$
$P_{15} = 0$
$P_{16} = 0$
$P_{17} = 0$
$P_{18} = 0$
$P_{19} = 0$
$P_{20} = 0$

17.7-1 (a) (i) exponential : $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

(ii) constant : $W_q = \frac{1}{2} \cdot \frac{\lambda}{\mu(\mu-\lambda)}$

(iii) Erlang : $\sigma = \frac{1}{2}(0 + \frac{1}{\mu}) = \frac{1}{2\mu} \Rightarrow \sigma^2 = \frac{1}{4\mu^2} \Rightarrow K = 4$

$$W_q = \frac{1+4}{8} \cdot \frac{\lambda}{\mu(\mu-\lambda)} = \frac{5}{8} \frac{\lambda}{\mu(\mu-\lambda)}$$

So $W_q^{exp} = 2 W_q^{c} = (8/5) W_q^{Erlang}$

(b) Let $B = 1, (1/2)$ and $(5/8)$ when the distribution is exponential, constant or Erlang, respectively.

Now $\lambda^{(2)} = 2\lambda^{(1)}$ and $\mu^{(2)} = 2\mu^{(1)}$

$$W_q^{(2)} = B\left[\frac{2\lambda^{(1)}}{2\mu^{(1)}(2\mu^{(1)} - 2\lambda^{(1)})}\right] = \frac{W_q^{(1)}}{2}$$

$$L_q^{(2)} = \lambda^{(2)} W_q^{(2)} = 2\lambda^{(1)} W_q^{(1)}/2 = \lambda^{(1)} W_q^{(1)} = L_q^{(1)}$$

So the waiting time is cut in half while the queue lenght is unchanged.

17.7-4 a)

Data

λ =	30	(mean arrival rate)
$1/\mu$ =	0.0208333	(expectec service time)
σ =	0.0208333	(standard deviation)
s =	1	(# servers)

Results

L =	1.66666667
L_q =	1.04166667
W =	0.05555556
W_q =	0.03472222

b)

Data

λ =	30	(mean arrival rate)
$1/\mu$ =	0.0208333	(expected service time)
σ =	0	(standard deviation)
s =	1	(# servers)

Results

L =	1.14583333
L_q =	0.52083333
W =	0.03819444
W_q =	0.01736111

c) L_q in part b) is half of L_q in part a).

d) Marsha needs to reduce her service time to approximately 61 seconds.

17.7-6 a & b) Current policy:

Data

$\lambda =$	1	(mean arrival rate)
$\mu =$	2	(mean service rate)
s =	1	(# servers)

Results

L =	1
L_q =	0.5
W =	1
W_q =	0.5

Proposal:

Data

$\lambda =$	0.25	(mean arrival rate)
$\mu =$	0.5	(mean service rate)
k =	4	(shape parameter)
s =	1	(# servers)

Results

L =	0.8125
L_q =	0.3125
W =	3.25
W_q =	1.25

Under the current policy an airplane looses 1 day of flying time as opposed to 3.25 days under the proposed policy.

Under the current policy 1 airplane is loosing flying time per day as opposed to 0.8125 airplanes.

c) The comparison in part b) is the appropriate one for making the decision since it takes into account that airplanes will not have to come in for service as often.

17.8-2

	W_{q1}	L_{q1}	W_1	L_1	W_{q2}	L_{q2}	W_2	L_2
$s=1$, $\mu=10$	.133	.533	.233	.933	.667	2.667	.767	3067
$s=2$, $\mu=5$	.119	.474	.319	1.274	.593	2.370	.793	3.170

If W_1 is the primary concern, one should choose the first alternative (one fast server). On the other hand, if W_{q1} is the primary concern, one should choose the second alternative (two slow servers)

17.8-5

$\lambda_1 = 0.1$, $\lambda_2 = 0.4$, $\lambda_3 = 1.5$, $\lambda = \sum_{i=1}^{3} \lambda_i = 2$, $\mu = 3$

	Preemptive Priorities		Nonpreemptive Priorities	
	$s=1$	$s=2$	$s=1$	$s=2$
A	...	...	4.5	36
B_1	.967	..	.967	.983
B_2	.833	...	.833	.917
B_3	.333	...	.333	.667
$W_1 - \frac{1}{\mu}$	.011	.00009	.230	.028
$W_2 - \frac{1}{\mu}$	.080	.00289	.276	.031
$W_3 - \frac{1}{\mu}$	.867	.05493	.800	.045