Solution Keys to Chapter 14 Problems

14.1(2) Label the products respectively A and B. Then the strategies for each manufacturer are:

1- Normal development of both products
2- Crash development of product A
3- Crash development of product B

Let $p_{ij} = \dfrac{(\%\text{ increase to I from A}) + (\%\text{ increase to I from B})}{2}$

The payoff matrix then becomes

I \ II	1	2	3	Row min
1	8	10	10	8 ← max
2	4	-4	13	-4
3	4	13	-4	-4
Column Maximum:	8 ↑ min	13	13	

Hence, both manufacturers should use normal development and I will increase his share of the market by 8%.

14.2(1) a) Strategies 4, 5 and 6 of player II are dominated by strategy 3.
Strategies 4, 5 and 6 of player I are dominated by strategy 3.
Strategy 1 of player II is dominated by strategy 3.
Strategy 1 of player I is dominated by strategy 3.
Strategy 2 of player II is dominated by strategy 3.
Strategy 2 of player I is dominated by strategy 3.
Therefore, the optimal strategy is for the labor union to decrease its demand by 20¢ and for management to increase its offer by 20¢.
A wage of $1.35 will be decided.

b)

I \ II	1	2	3	4	5	6	Row Minimum
1	1.35	1.2	1.3	1.4	1.5	1.6	1.2
2	1.5	1.35	1.3	1.4	1.5	1.6	1.3
3	1.4	1.4	1.35	1.4	1.5	1.6	1.35 ← max
4	1.3	1.3	1.3	1.35	1.5	1.6	1.3
5	1.2	1.2	1.2	1.2	1.35	1.6	1.2
6	1.1	1.1	1.1	1.1	1.1	1.35	1.1
Column Maximum	1.5	1.4	1.35 ↑ min	1.4	1.5	1.6	

14.2(2)

(a) Strategy 3 of player I is dominated by strategy 2.
Strategy 3 of player II is dominated by strategy 1.
Strategy 1 of player I is dominated by strategy 2.
Strategy 2 of player II is dominated by strategy 1.
Therefore, the optimal strategy is for player I to choose strategy 2 and player II to choose strategy 1 resulting in a payoff of 1 to player I.

b) Strategy 1 of player II is dominated by strategy 3.
Strategy 2 of player I is dominated by both strategies 1 and 3.
Strategy 2 of player II is dominated by strategy 3.
Strategy 1 of player I is dominated by strategy 3.
Therefore, the optimal strategy is for player I to choose strategy 3 and player II to choose strategy 3 resulting in a payoff of 1 to player II.

14.2(4)

		II 1	2	3	Row Minimum
	1	1	-1	1	-1
I	2	-2	0	3	-2
	3	3	1	2	1 ← max
Column Maximum		3	1 ↑ min	3	

v = 1, player I uses strategy 3 and player II uses strategy 2.

The game is stable with saddlepoint (3,2)

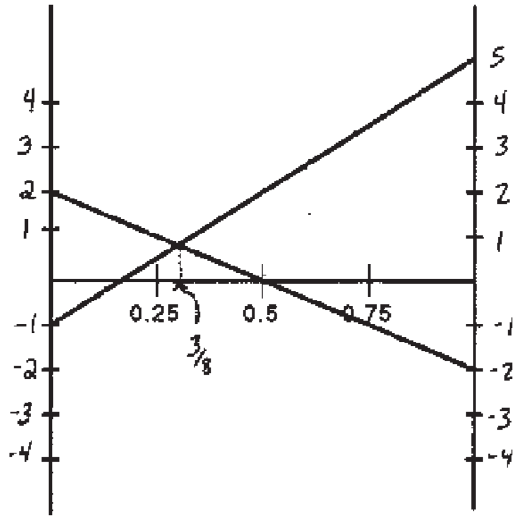
14.4(2)

(y_1, y_2)	Expected Payoff
$(1,0)$	$3x_1 - 1(1-x_1) = 4x_1 - 1$
$(0,1)$	$-2x_1 + 2(1-x_1) = -4x_1 + 2$

$$4x_1 - 1 = -4x_1 + 2 \Rightarrow x_1^* = 3/8 \,,\; x_2^* = 5/8$$

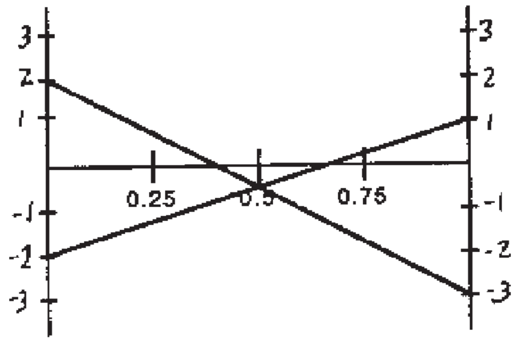
$$v = 4(3/8) - 1 = 1/2$$

$$\left.\begin{array}{l} 3y_1^* - 2y_2^* = 1/2 \\ -y_1^* + 2y_2^* = 1/2 \end{array}\right\} \Rightarrow y_1^* = y_2^* = 1/2$$



The payoff matrix for player II is:

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{II}} \\ & & 1 & 2 \\ \hline \text{I} & 1 & -3 & 2 \\ & 2 & 1 & -2 \end{array}$$



(x_1, x_2)	Expected Payoff
$(1,0)$	$-3y_1 + 2(1-y_1) = -5y_1 + 2$
$(0,1)$	$y_1 - 2(1-y_1) = 3y_1 - 2$

$$-5y_1 + 2 = 3y_1 - 2 \Rightarrow y_1^* = \tfrac{1}{2} \,,\; y_2^* = 1/2$$

14.5(4)

1) To insure $x_5 \geq 0$ add 4 to each entry of the payoff table.

Maximize

subject to:

$$5x_1 + 6x_2 + 4x_3 - x_5 \geq 0$$
$$x_1 + 7x_2 + 8x_3 + 4x_4 - x_5 \geq 0$$
$$6x_1 + 4x_2 + 3x_3 + 2x_4 - x_5 \geq 0$$
$$2x_1 + 7x_2 + x_3 + 6x_4 - x_5 \geq 0$$
$$5x_1 + 2x_2 + 6x_3 + 3x_4 - x_5 \geq 0$$
$$x_1 + x_2 + x_3 + x_4 = 1$$

$x_i \geq 0$ for $i = 1,2,3,4,5$

Solve Automatically by the Simplex Method:

1) **Optimal Solution**

Value of the
Objective Function: Z = 3.98101266

Variable	Value
X_1	0.31013
X_2	0.26582
X_3	0.20886
X_4	0.21519
X_5	3.98101

14.2(6)(a)

$$\begin{array}{cc|ccc|c} & & & \text{II} & & \text{Row} \\ & & 1 & 2 & 3 & \text{Minimum} \\ \hline & 1 & 2 & 3 & 1 & 1 \leftarrow \text{max} \\ \text{I} & 2 & 1 & 4 & 0 & 0 \\ & 3 & 3 & -2 & -1 & -2 \\ \hline \text{Column maximum} & & 3 & 4 & \underset{\text{min}}{\underset{\uparrow}{1}} & \end{array}$$

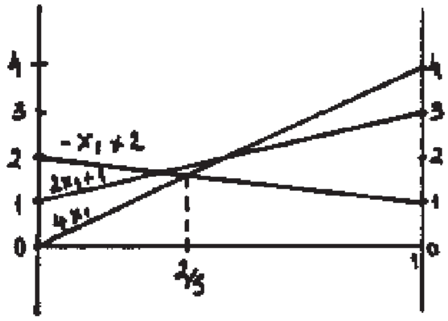
$v = 1$, player I uses strategy 1 and player II uses strategy 3.

(b) Strategy 1 of player II is dominated by strategy 3.
Strategy 3 of player I is dominated by strategies 1 and 2.
Strategy 2 of player II is dominated by strategy 3.
Strategy 2 of player I is dominated by strategy 1.
Therefore, the optimal strategy is for player I to choose strategy 1 and player II to choose strategy 3 resulting in a payoff of 1 to player I.

14.4(3)

(a)

(y_1, y_2, y_3)	Expected payoff
(1, 0, 0)	$4x_1$
(0, 1, 0)	$3x_1 + (1-x_1) = 2x_1 + 1$
(0, 0, 1)	$x_1 + 2(1-x_1) = -x_1 + 2$



$4x_1 = -x_1 + 2 \Rightarrow (x_1^*, x_2^*) = (2/5, 3/5)$

and $v = 8/5$

$y_1^*(4x_1) + y_3^*(-x_1+2) = 8/5$ for $0 \le x_1 \le 1 \Rightarrow 2y_3^* = 8/5$

$4y_1^* + y_3^* = 8/5$

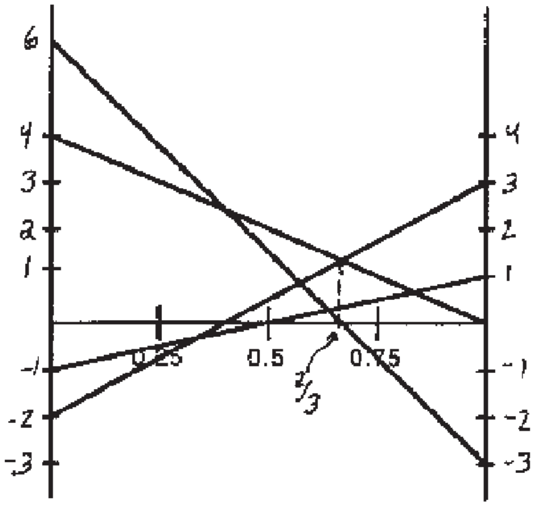
$\Rightarrow (y_1^*, y_2^*, y_3^*) = (1/5, 0, 4/5)$

(b)

Strategy 3 of player II is dominated by strategy 1 reducing the table to:

		II 1	2
I	1	1	-1
	2	0	4
	3	3	-2
	4	-3	6

(x_1, x_2, x_3, x_4)	Expected Payoff
(1, 0, 0, 0)	$y_1 - (1-y_1) = 2y_1 - 1$
(0, 1, 0, 0)	$4(1-y_1) = -4y_1 + 4$
(0, 0, 1, 0)	$3y_1 - 2(1-y_1) = 5y_1 - 2$
(0, 0, 0, 1)	$-3y_1 + 6(1-y_1) = -9y_1 + 6$



$y_1^* = 2/3 \quad y_2^* = 1/3$

$v = 4/3$

Let $x^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ be the optimal solu. s.t.

$$\frac{4}{3} = v = \underline{v} = \bar{v} = E(x^*, y^*)$$
$$= x_1^* \ell_1(y_1^*) + x_2^* \ell_2(y_1^*) + x_3^* \ell_3(y_1^*) + x_4^* \ell_4(y_1^*)$$

with

$$\ell_1(y_1) = 2y_1 - 1, \quad \ell_2(y_1) = -4y_1 + 4, \quad \ell_3(y_1) = 5y_1 - 2, \quad \ell_4(y_1) = -9y_1 + 6$$

and $\ell_1(y_1^*) = \frac{1}{3} < \frac{4}{3} = \bar{v}$, $\ell_2(y_1^*) = \frac{4}{3} = \bar{v}$, $\ell_3(y_1^*) = \frac{4}{3} = \bar{v}$, $\ell_4(y_1^*) < \bar{v}$.

$\Rightarrow \bar{v} = E(x^*, y^*)$ must force $x_1^* = x_4^* = 0$ because

Otherwise if $x_1^* > 0$ or $x_4^* > 0$,

$$E(x^*, y^*) < (x_1^* + x_2^* + x_3^* + x_4^*)\bar{v} = 1 \cdot \bar{v} = \bar{v}$$

a contradiction!

Since
$$\bar{v} = \min_{\substack{0 \le y_1 \le 1 \\ y_2 = 1 - y_2}} E(x^*, y) \le E(x^*, y) = x_2^*(-4y_1 + 4) + x_3^*(5y_1 - 2)$$

for $0 \le y_1 \le 1$; $E(x^*, y^*) = \bar{v}$ with $y_1 = y_1^*$; and since $E(x^*, y)$ is linear in y_1, we must have $E(x^*, y)$ to be a constant function in y_1. Therefore

$$E(x^*, y) = x_2^*(-4y_1 + 4) + x_3^*(5y_1 - 2) \equiv \bar{v} = \frac{4}{3}$$

In particular, at $y_1 = 0$, $E(x^*, y) = 4x_2^* - 2x_3^* = \frac{4}{3}$.

Since $x_2^* + x_3^* = 1$, solving $\begin{cases} 4x_2^* - 2x_3^* = \frac{4}{3} \\ x_2^* + x_3^* = 1 \end{cases}$ gives

$$x_2^* = \frac{5}{9}, \quad x_3^* = \frac{4}{9}.$$

Optimal solu: $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, \frac{5}{9}, \frac{4}{9}, 0)$

$(y_1^*, y_2^*) = (2/3, 1/3)$

Exp. payoff/game value: $v = \underline{v} = \bar{v} = 4/3$

14.5(2)

a) Maximize x_4

subject to
$$
\begin{aligned}
5x_1+2x_2+3x_3-x_4 &\geq 0\\
4x_2+2x_3-x_4 &\geq 0\\
3x_1+3x_2 \quad -x_4 &\geq 0\\
x_1+2x_2+4x_3-x_4 &\geq 0\\
x_1+x_2+x_3 &= 1\\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}
$$

b) Solve Automatically by the Simplex Method:

Optimal Solution

Value of the
Objective Function: Z = 2.36842105

Variable	Value
X_1	0.05263
X_2	0.73684
X_3	0.21053
X_4	2.36842

14.5-8 AUTOMATIC SIMPLEX METHOD: FINAL TABLEAU

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Right side
									1M	1M	1M	1M	
Z	0	1	0	0	0	0	0.455	0.545	0	-0.45	-0.55	0.182	0.182
X2	1	0	0	1	0	0	-0.09	0.091	0	0.091	-0.09	0.364	0.364
X4	2	0	0	0	0	1	-0.91	-0.09	-1	0.909	0.091	1.636	1.636
X1	3	0	1	0	0	0	0.091	-0.09	0	-0.09	0.091	0.636	0.636
X3	4	0	0	0	1	0	0.455	0.545	0	-0.45	-0.55	0.182	0.182

The optimal primal solution is $(x_1, x_2) = (0.636, 0.364)$ with a payoff = 0.182

The optimal dual solution is $(y_1, y_2, y_3) = (0, 0.455, 0.545)$