

e)

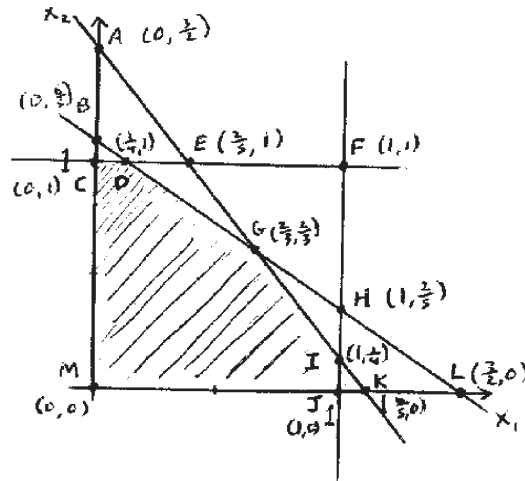
Corner Point	Profit = $3x_1 + 2x_2$	Next Step
D: (0,0)	0	Check points A and C.
A: (0,3)	6	Move to C.
C: (3,0)	9	Check point B.
B: (2,2)	10	Stop, (2,2) is optimal. *

* the next corner point is A, which has already been checked

4.1-4 a), b)

	CP sol'n	Feasible?	Value
A	$(0, \frac{3}{2})$	Infeas.	6750
B	$(0, \frac{5}{2})$	Infeas.	5400
C	$(0, 1)$	Feas.	4500
D	$(\frac{1}{4}, 1)$	F	5625
E	$(\frac{2}{3}, 1)$	I	6300
F	$(1, 1)$	I	9000
G	$(\frac{2}{3}, \frac{2}{3})$	F	6000 *
H	$(1, \frac{2}{3})$	I	6300
I	$(1, \frac{1}{4})$	F	5625
J	$(1, 0)$	F	4500
K	$(\frac{4}{3}, 0)$	I	5400
L	$(\frac{3}{2}, 0)$	I	6750
M	$(0, 0)$	F	0

* optimal



c) Initial Pt M $[(0,0)] \rightarrow$ Not optimal since C and J $[(0,1) \text{ and } (1,0)]$ are better.

Since C and J give same objective value, either can be chosen.

We will choose C.

C is not optimal since D $[(\frac{1}{4}, 1)]$ is adjacent and better. Move to D.

(Note that B is CP sol'n but not feasible)

D is not optimal since G $[(\frac{2}{3}, \frac{2}{3})]$ is adjacent and better. Move to G.

(Note that G is the only better adjacent CPF sol'n, D is CPF, but worse.)

G is optimal since CPF's D and I give objective values $5625 < 6000$.

Note: If J is chosen at first step, the path will be: M-J-I-G
(with the same arguments as above).

4.2-3. a) augmented form:

$$\begin{aligned} \text{Max } & 3x_1 + 2x_2 \\ \text{s.t. } & x_1 + x_3 = 4 \\ & x_1 + 3x_2 + x_4 = 15 \\ & 2x_1 + x_2 + x_5 = 10 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0 \end{aligned}$$

b)

	CPF Sol'n	BF Sol'n	Non-basic Var's	Basic Var's
A	(0,0)	(0,0,4,15,10)	x_1, x_2	x_3, x_4, x_5
B	(0,5)	(0,5,4,0,5)	x_1, x_4	x_2, x_3, x_5
C	(3,4)	(3,4,1,0,0)	x_4, x_5	x_1, x_2, x_3
D	(4,2)	(4,2,0,5,0)	x_3, x_5	x_1, x_2, x_4
E	(4,0)	(4,0,0,11,2)	x_2, x_3	x_1, x_4, x_5

c)

BF Sol'n	Set Non-basic Var's to zero	Solve:
A	$x_1 = x_2 = 0$	$x_3 = 4, x_4 = 15, x_5 = 10$
B	$x_1 = x_4 = 0$	$\begin{cases} x_3 = 4 \\ 3x_2 = 15 \\ x_2 + x_5 = 10 \end{cases} \Rightarrow \begin{cases} x_2 = 5 \\ x_5 = 10 - 5 = 5 \end{cases}$
C	$x_4 = x_5 = 0$	$\begin{cases} x_1 + x_3 = 4 \\ x_1 + 3x_2 = 15 \\ 2x_1 + x_2 = 10 \end{cases} \Rightarrow \begin{cases} 2x_1 + 6x_2 = 30 \\ 2x_1 + x_2 = 10 \\ \hline 5x_2 = 20 \end{cases} \Rightarrow \begin{cases} x_2 = 4 \\ x_1 = 3 \\ x_3 = 4 - 3 = 1 \end{cases}$
D	$x_3 = x_5 = 0$	$\begin{cases} x_1 = 4 \\ x_1 + 3x_2 + x_4 = 15 \\ 2x_1 + x_2 = 10 \end{cases} \Rightarrow \begin{cases} x_2 = 10 - 2(4) = 2 \\ x_4 = 15 - 4 - 3(2) = 5 \end{cases}$
E	$x_2 = x_3 = 0$	$\begin{cases} x_1 = 4 \\ x_1 + x_4 = 15 \\ 2x_1 + x_5 = 10 \end{cases} \Rightarrow \begin{cases} x_4 = 15 - 4 = 11 \\ x_5 = 10 - 2(4) = 2 \end{cases}$

-4.3-2-

$$\begin{aligned} \text{a) } \max Z &= x_1 + 2x_2 \\ \text{s.t. } x_1 + 3x_2 + x_3 &= 8 \\ x_1 + x_2 + x_4 &= 4 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \end{aligned}$$

Initialization : $x_1 = x_2 = 0 \Rightarrow x_3 = 8, x_4 = 4$

$$Z = x_1 + 2x_2 \Rightarrow Z = 0, \text{ not optimal since improvement rates } > 0$$

Increase x_2 (rate of improvement = 2)

↑ "entering basic variable for iteration 1"

$$x_1 = 0, \text{ so } x_3 = 8 - 3x_2 \geq 0 \Rightarrow x_2 \leq \frac{8}{3} \leftarrow \min$$

$$x_4 = 4 - x_2 \geq 0 \Rightarrow x_2 \leq 4$$

$$\text{Thus, } x_2 \text{ can be increased to } \frac{8}{3} \Rightarrow x_3 = 0$$

↑ "leaving basic variable for iteration 1"

Using Gaussian elimination, we get:

$$Z = \frac{1}{3}x_1 - \frac{2}{3}x_3 + \frac{16}{3}$$

$$\frac{1}{3}x_1 + x_2 + \frac{1}{3}x_3 = \frac{8}{3}$$

$$\frac{2}{3}x_1 - \frac{1}{3}x_3 + x_4 = \frac{4}{3} \quad (x_1, x_2, x_3, x_4 \geq 0)$$

Again $(0, \frac{8}{3}, 0, \frac{4}{3})$ is not optimal since rate of improvement for x_1 is > 0 .

Increase x_1 , ($x_3 = 0$)

$$x_2 = \frac{8}{3} - \frac{1}{3}x_1 \geq 0 \Rightarrow x_1 \leq 8$$

$$x_4 = \frac{4}{3} - \frac{2}{3}x_1 \geq 0 \Rightarrow x_1 \leq 2 \leftarrow \min$$

$$x_1 \text{ can be increased to } 2 \Rightarrow x_4 = 0$$

43-2a) (cont')

Again, using Gaussian elimination:

$$Z = -\frac{1}{2}x_3 - \frac{1}{2}x_4 + 6$$

$$x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_4 = 2$$

$$x_1 - \frac{1}{2}x_3 + \frac{3}{2}x_4 = 2 \quad (x_1, x_2, x_3, x_4 \geq 0)$$

Now, we are optimal since increasing x_3 or x_4 would decrease Z .

$$x^* = (2, 2, 0, 0), \quad Z^* = 6 \quad (\text{ie. } x_1^* = 2, x_2^* = 2)$$

b) Computer output:

Solve Interactively by the Simplex Method:

$$\begin{array}{l} 0) \quad Z - 1x_1 - 2x_2 + 0x_3 + 0x_4 = 0 \\ 1) \quad -x_1 + 3x_2 + 1x_3 + 0x_4 = 8 \\ 2) \quad 1x_1 + 1x_2 + 0x_3 + 1x_4 = 4 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

$$\begin{array}{l} 0) \quad Z - 0.333x_1 + 0x_2 + 0.67x_3 + 0x_4 = 5.33333 \\ 1) \quad 0.333x_1 + 1x_2 + 0.33x_3 + 0x_4 = 2.66667 \\ 2) \quad 0.667x_1 + 0x_2 - 0.33x_3 + 1x_4 = 1.33333 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

$$\begin{array}{l} 0) \quad Z + 0x_1 + 0x_2 + 0.5x_3 + 0.5x_4 = 6 \\ 1) \quad 0x_1 + 1x_2 + 0.5x_3 - 0.5x_4 = 2 \\ 2) \quad 1x_1 + 0x_2 - 0.5x_3 + 1.5x_4 = 2 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

$$x_1^* = x_2^* = 2, \quad Z^* = 6$$

c) Again, you get the same solution:

Optimal Solution

Value of the
Objective Function: $Z = 6$

Variable	Value
x_1	2
x_2	2

Sensitivity Analysis

Objective Function Coefficient

Current Value	Allowable Range To Stay Optimal	
	Minimum	Maximum
1	0.66667	2
2	1	3

- 45-1. a) True. The Ratio test tells us how far we can increase the entering basic variable before one of the current basic variables goes to zero. If two variables "tie" for which should be the leaving variable, then both variables go to zero at the same value of the entering basic variable. Since only one variable can become non-basic in any iteration, the other will be basic and zero.
- b) False. If there is no leaving basic variable, then the solution is unbounded. (Can increase the entering basic variable indefinitely)
- c) True, usually. If we pivot on the column with the zero coefficient, then we will get another optimal solution (remember that the objective value will not change since the "rate of improvement" is zero. The exception occurs when the problem is also degenerate. If the basic variable (which has a coefficient of zero) is itself equal to zero (i.e. degenerate), then pivoting on that column will not change the optimal solution.

d) False. Examples; (i) $\text{Max } x_1 - x_2$

$$x_1 - x_2 \leq 1$$

$$x_1 \geq 0, x_2 \geq 0$$

Clearly, $z^* = 1$ (optimal) but

$$x_1^* = k+1, x_2^* = k \quad k \in [0, \infty)$$

(ii) $\text{Max } -x_1$

$$x_1 + x_2 \geq -1$$

$$x_1 \geq 0, x_2 \geq 0$$

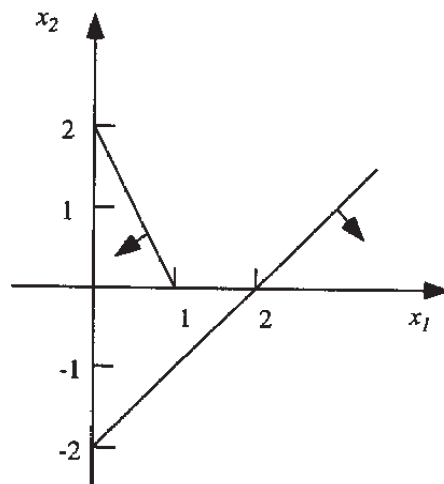
This is a trivial

example, $x_1^* = 0$

$$x_2^* \geq 0, z^* = 0$$

4.6-6

a)



b)

Constraint	Contribution Per Unit of Each Activity		Totals		Resource
	Activity 1	Activity 2			
1	2	1	2	$\leq$	2
2	1	-1	1	$\geq$	2
Unit Profit	90	70	\$ 90		
Solution	1	0			

Solver could not find a feasible solution.

c)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
10)	Z	0	1	-90	-70	1.0e6	0	1.0e6	0
	X1	1	0	2*	1	1	0	0	2
	X1	2	0	1	-1	0	-1	1	2

ii)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
ii)	Z	0	1	0	-25	1.0e6	0	1.0e6	90
	X1	1	0	1	0.5	0.5	0	0	1
	X1	2	0	0	-1.5	-0.5	-1	1*	1

ii)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
ii)	Z	0	1	0	1.5e6	1.5e6	1.0e6	0	-1e6
	X1	1	0	1	0.5	0.5	0	0	1
	X5	2	0	0	-1.5	-0.5	-1	1	1

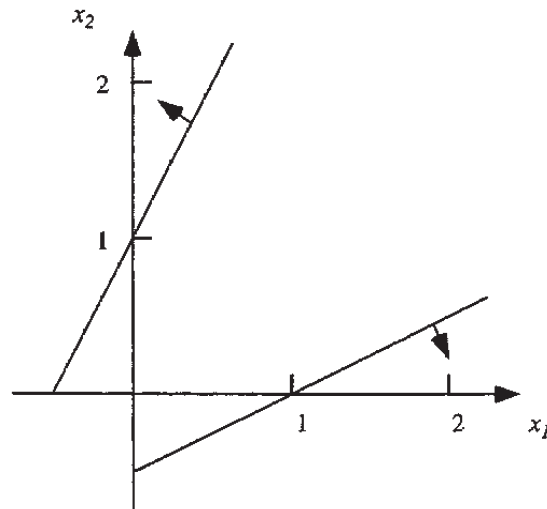
d)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
10)	Z	0	1	0	0	1	0	1	0
	X1	1	0	2*	1	1	0	0	2
	X1	2	0	1	-1	0	-1	1	2

ii)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
ii)	Z	0	1	0	0	1	0	1	0
	X1	1	0	1	0.5	0.5	0	0	1
	X1	2	0	0	-1.5	-0.5	-1	1*	1

(1)	Bas Var	Eq No	Z	Coefficient of					Right side
				X1	X2	X3	X4	X5	
(1)	Z	0	1	0	1.5	1.5	1	0	-1
	X1	1	0	1	0.5	0.5	0	0	1
	X5	2	0	0	-1.5	-0.5	-1	1	1

4.6-7

a)



b)

Benefit	Benefit Contribution Per Unit of Each Activity		Totals		Minimum Level
	Activity 1	Activity 2			
1	-2	1	0	≥	1
2	1	-2	0	≥	1
Unit Cost	5000	7000	\$	-	
Solution	0	0			

Solver could not find a feasible solution.

c)

Bas Var	Eq No	Z	Coefficient of						Right side
			X1	X2	X3	X4	X5	X6	
Z	0	1	5000	7000	0	0	1.0e6	1.0e6	0
X1	1	0	-2	1	-1	0	1	0	1
X1	2	0	1*	-2	0	-1	0	1	1

Bas Var	Eq No	Z	Coefficient of						Right side
			X1	X2	X3	X4	X5	X6	
Z	0	1	0	17000	0	5000	1.0e6	1.0e6	-5000
X1	1	0	0	-3	-1	-2	1	2	3
X1	2	0	1	-2	0	-1	0	1*	1

Bas Var	Eq No	Z	Coefficient of						Right side
			X1	X2	X3	X4	X5	X6	
Z	0	1	-1e6	2.0e6	0	1.0e6	1.0e6	0	-1e6
X1	1	0	-2	1	-1	0	1*	0	1
X6	2	0	1	-2	0	-1	0	1	1

(Co NTP 7)

(cont'd)

	Bas Var	Eq No	Z	Coefficient of						Right side
				X1	X2	X3	X4	X5	X6	
(3)	Z	0	1	1.0e6	1.0e6	1.0e6	1.0e6	0	0	-2e6
	X5	1	0	-2	1	-1	0	1	0	1
	X6	2	0	1	-2	0	-1	0	1	1

d)

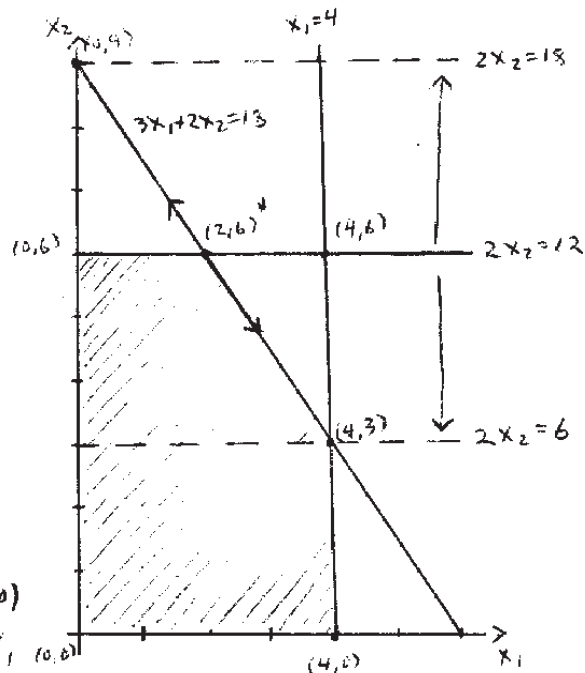
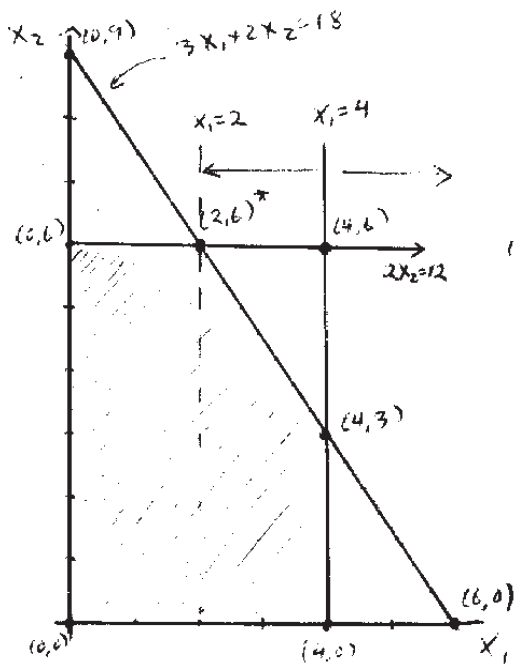
	Bas Var	Eq No	Z	Coefficient of						Right side
				X1	X2	X3	X4	X5	X6	
(v)	Z	0	1	0	0	0	0	1	1	0
	X1	1	0	-2	1	-1	0	1	0	1
	X1	2	0	1*	-2	0	-1	0	1	1

	Bas Var	Eq No	Z	Coefficient of						Right side
				X1	X2	X3	X4	X5	X6	
(i)	Z	0	1	0	0	0	0	1	1	0
	X1	1	0	0	-3	-1	-2	1*	2	3
	X1	2	0	1	-2	0	-1	0	1	1

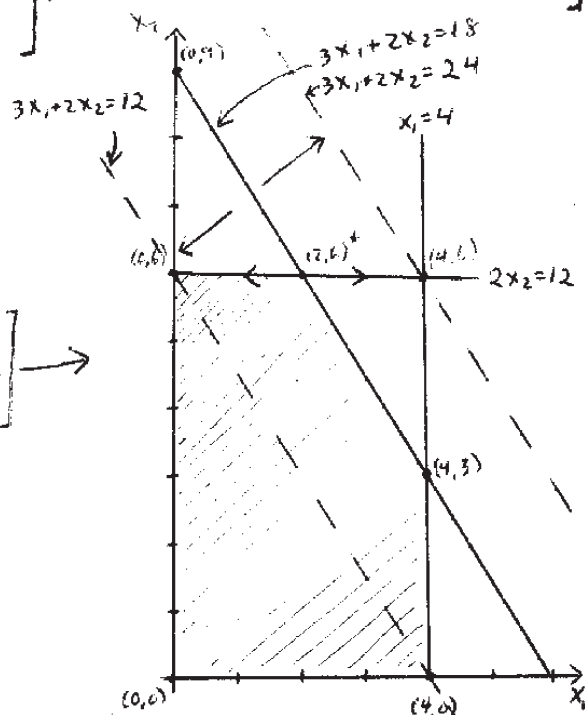
	Bas Var	Eq No	Z	Coefficient of						Right side
				X1	X2	X3	X4	X5	X6	
(2)	Z	0	1	0	3	1	2	0	-1	-3
	X5	1	0	0	-3	-1	-2	1	2	3
	X1	2	0	1	-2	0	-1	0	1*	1

	Bas Var	Eq No	Z	Coefficient of						Right side
				X1	X2	X3	X4	X5	X6	
(3)	Z	0	1	1	1	1	1	0	0	-2
	X5	1	0	-2	1	-1	0	1	0	1
	X6	2	0	1	-2	0	-1	0	1	1

4.7-1.



The CP Sol'n $(2,6)$ remains feasible for $x_1 = k$ $2 \leq k < \infty$. For $k < 2$, that CP Sol'n become infeasible. $(2,6)$ is, of course, the optimal.



The CP Sol'n $(\frac{k}{3} - 4, 6)$ remains feasible for $3x_1 + 2x_2 = k$ $12 \leq k \leq 24$. (Infeasible for other values of k)

4.3.2. (CONT'D)

Looking at the "—" lines, the CP Soln ranges from (4,0) to (0,4) as b_1 ranges from 4 to 12. The CP Soln becomes infeasible for values of b_1 outside of this range.

Looking at the "—" lines, the CP Soln ranges from (0, $\frac{8}{3}$) to (0, 8) as b_2 ranges from $\frac{8}{3}$ to 8.

The allowable ranges are, therefore,

$$4 \leq b_1 \leq 12$$

$$\text{and } \frac{8}{3} \leq b_2 \leq 8$$

c)

Variables	1	2	6	Optimal Value
	2	2		
Constraints	1	3	8 <=	8
	1	1	4 <=	4

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$2		2	0	1	1	0.333333
\$C\$2		2	0	2	1	1

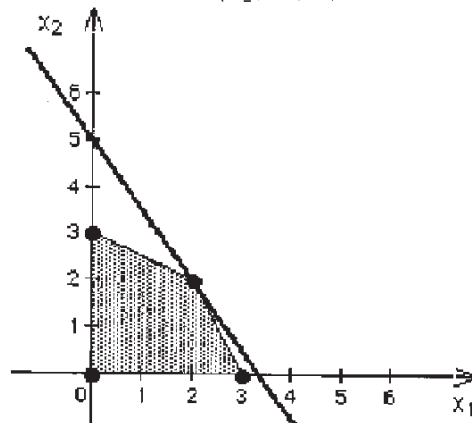
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$4		8	0.5	8	4	4
\$E\$5		4	0.5	4	4	1.333333

Solution Keys to Chapter 4 Assignment

4.1-2a)

a) Optimal Solution: $(x_1, x_2) = (2, 2)$ and Profit = 10.



Note: corner points will be called A, B, C, and D going clockwise from (0,3).

b)

Corner Point	Corresponding Constraint Boundary Equations	Corner Point Satisfies the Equations
A: (0,3)	$x_1 = 0$ $x_1 + 2x_2 = 6$	$0=0$ $0+2(3)=6$
B: (2,2)	$x_1 + 2x_2 = 6$ $2x_1 + x_2 = 6$	$2+2(2)=6$ $2(2)+2=6$
C: (3,0)	$2x_1 + x_2 = 6$ $x_2 = 0$	$2(3)+0=6$ $0=0$
D: (0,0)	$x_2 = 0$ $x_1 = 0$	$0=0$ $0=0$

c)

Corner Point	Its Adjacent Corner Points
A: (0,3)	(0,0) and (2,2)
B: (2,2)	(0,3) and (3,0)
C: (3,0)	(2,2) and (0,0)
D: (0,0)	(3,0) and (0,3)

d)

Corner Point (x_1, x_2)	Profit = $3x_1 + 2x_2$
(0,3)	6
(2,2)	10
(3,0)	9
(0,0)	0

Optimal Solution: $(x_1, x_2) = (2, 2)$ and Profit = 10.

e)

Corner Point	Profit = $3x_1 + 2x_2$	Next Step
D: (0,0)	0	Check points A and C.
A: (0,3)	6	Move to C.
C: (3,0)	9	Check point B.
B: (2,2)	10	Stop, (2,2) is optimal. *

* the next corner point is A, which has already been checked

4.3-5 (OUTPUT)

Solve Interactively by the Simplex Method:

$$\begin{array}{lcl} 0) & z - & 1 \cdot x_1 - 2 \cdot x_2 - 4 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6 = 0 \\ 1) & & 3 \cdot x_1 + 1 \cdot x_2 + 5 \cdot x_3 + 1 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6 = 10 \\ 2) & & 1 \cdot x_1 + 4 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 + 1 \cdot x_5 + 0 \cdot x_6 = 8 \\ 3) & & 2 \cdot x_1 + 0 \cdot x_2 + 2 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 + 1 \cdot x_6 = 7 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0.$$

$$\begin{array}{lcl} \text{C)} & \text{Z} + 1.4 \text{ X}_1 - 1.2 \text{ X}_2 + & 0 \text{ X}_3 + 0.8 \text{ X}_4 + 0 \text{ X}_5 + 0 \text{ X}_6 = 8 \\ \text{1)} & 0.6 \text{ X}_1 + 0.2 \text{ X}_2 + & 1 \text{ X}_3 + 0.2 \text{ X}_4 + 0 \text{ X}_5 + 0 \text{ X}_6 = 2 \\ \text{2)} & 0.4 \text{ X}_1 + 3.8 \text{ X}_2 + & 0 \text{ X}_3 - 0.2 \text{ X}_4 + 1 \text{ X}_5 + 0 \text{ X}_6 = 6 \\ \text{3)} & 0.8 \text{ X}_1 - 0.4 \text{ X}_2 + & 0 \text{ X}_3 - 0.4 \text{ X}_4 + 0 \text{ X}_5 + 1 \text{ X}_6 = 3 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0.$$

0)	$Z+1.53$	X_1+	0	X_2+	0	$X_3+0.74$	$X_4-0.32$	X_5+	0	$X_6 = 9.89474$
1)	0.579	X_1+	0	X_2+	1	$X_3+0.21$	$X_4-0.05$	X_5+	0	$X_6 = 1.68421$
2)	0.105	X_1+	1	X_2+	0	$X_3-0.05$	$X_4-0.26$	X_5+	0	$X_6 = 1.57895$
3)	0.842	X_1+	0	X_2+	0	$X_3-0.42$	$X_4-0.11$	X_5+	1	$X_6 = 3.63158$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0.$$

Optimal solution is : $(0, \frac{30}{19}, \frac{32}{19}) = (x_1^*, x_2^*, x_3^*)$, $z^* = 9\frac{17}{19}$

4.3.8

(a) Because $x_2 = 0$ in the optimal solution, we need only solve the linear program:

Maximize $z = 2x_1 + 3x_3$

subject to: $x_1 + 2x_2 \leq 30$

$$x_1 + x_3 \leq 24$$

$$3x_1 + 3x_2 \leq 60$$

$$x_1 \geq 0, x_2 \geq 0.$$

or, equivalently,

Maximize $z = 2x_1 + 3x_3$

subject to: $x_1 + 2x_3 \leq 30$

$$x_1 + x_3 \leq 20$$

$$x_1 \geq 0 \quad x_2 \geq 0.$$

Since $x_1 > 0$ and $x_3 > 0$ in the optimal solution, they must be basic variables in the optimal solution. Choosing x_1 and x_3 as the first two entering variables will lead directly to the optimal solution. Leaving variables must still be determined by a minimum ratio test.

(b)

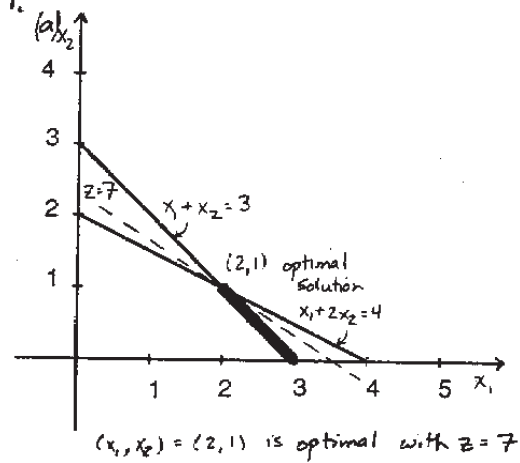
Bas Var	Eq		Coefficient of				Right side
	No	Z	X1	X3	X4	X5	
Z	0	1	-2	-3	0	0	0
X4	1	0	1	2*	1	0	30
X5	2	0	1	1	0	1	20

Bas Var	Eq No	Z	Coefficient of				Right side
			X1	X3	X4	X5	
① Z	0	1	-0.5	0	1.5	0	45
X3	1	0	0.5	1	0.5	0	15
X5	2	0	0.5*	0	-0.5	1	5

Bas Var	Eq		Coefficient of				Right side
	No	Z	X1	X3	X4	X5	
(2) Z	0	1	0	0	1	1	50
X3	1	0	0	1	1	-1	10
X1	2	0	1	0	-1	2	10

$(x_1, x_2, x_3) = (10, 0, 10)$ is optimal
with $z = 50$

4.6-1.



b)

(0)

Bas Var	Eq No	Z	Coefficient of				Right Side
			x_1	x_2	x_3	x_4	
			-1M	-1M			
Z	0	1	-2	-3	0	0	-3M
x_3	1	0	1	2	1	0	4
x_4	2	0	1	1	0	1	3

The initial artificial basic feasible solution is $(x_1, x_2, x_3, x_4) = (0, 0, 4, 3)$.

c)

(1)

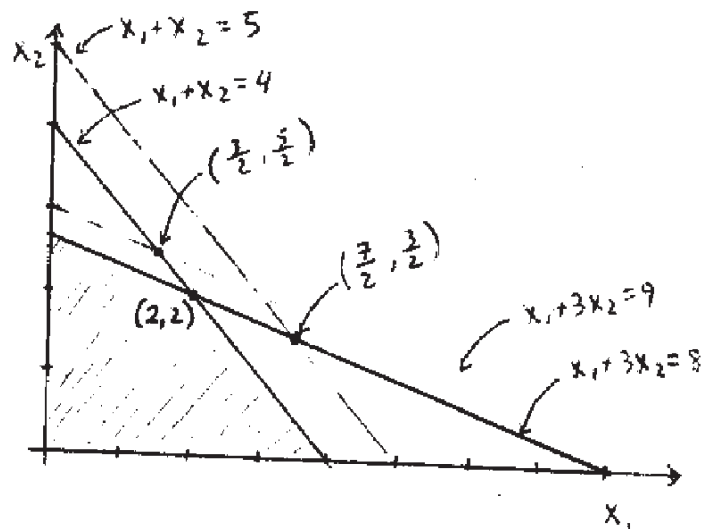
Bas Var	Eq No	Z	Coefficient of				Right Side
			x_1	x_2	x_3	x_4	
			-0.5M		0.5M		-1M
Z	0	1	-0.5	0	+1.5	0	+6
x_2	1	0	0.5	1	0.5	0	2
x_4	2	0	0.5	0	-0.5	1	1

(2)

Bas Var	Eq No	Z	Coefficient of				Right Side
			x_1	x_2	x_3	x_4	
						1M	
Z	0	1	0	0	1	+1	7
x_2	1	0	0	1	1	-1	1
x_1	2	0	1	0	-1	2	2

Optimal; $(x_1^*, x_2^*) = (2, 1)$, $z^* = 7$

4.7.2. a)



Constraint 1 ("---" line):

$$\left. \begin{array}{l} x_1 + 3x_2 = 8 \rightarrow Z = (2) + 2(2) = 6 \\ x_1 + 3x_2 = 9 \rightarrow Z = (\frac{3}{2}) + 2(\frac{5}{2}) = \frac{13}{2} \end{array} \right\} \Delta Z = \frac{1}{2} = Y_1^*$$

Constraint 2 ("— · — · —" line)

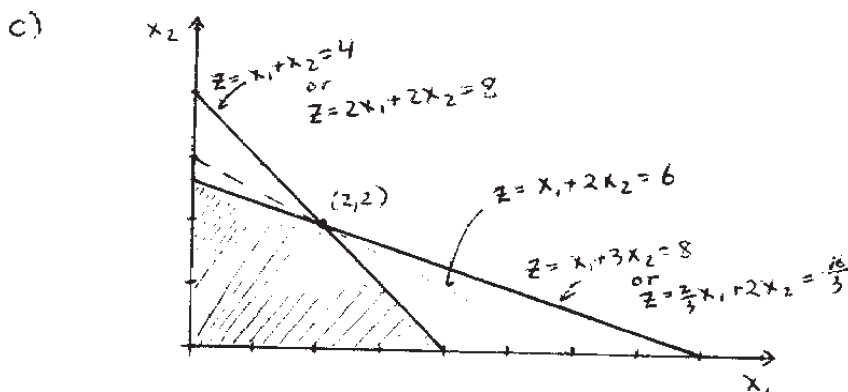
$$\left. \begin{array}{l} x_1 + x_2 = 4 \rightarrow Z = (2) + 2(2) = 6 \\ x_1 + x_2 = 5 \rightarrow Z = (\frac{3}{2}) + 2(\frac{7}{2}) = \frac{13}{2} \end{array} \right\} \Delta Z = \frac{1}{2} = Y_2^*$$

4.7-2

b) From (a), we see that $b_1 = 8$, $b_2 = 4$ are sensitive parameters.

As will be seen in part (c), $c_1 = 1$, $c_2 = 2$ are not sensitive parameters.

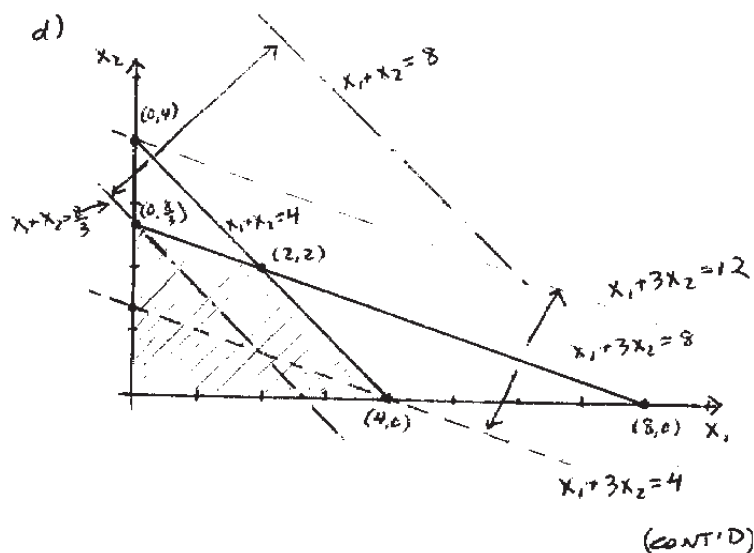
As can be seen in the figure in part (a), both constraints are active or binding, so all coefficients $a_{11} = 1$, $a_{12} = 3$, $a_{21} = 1$, $a_{22} = 1$ are sensitive parameters.



We see that the optimal solution remains the same for:

$$\frac{2}{3} \leq c_1 \leq 2 \quad (c_2 \text{ fixed at } 2)$$

$$\text{and } 1 \leq c_2 \leq 3 \quad (c_1 \text{ fixed at } 1)$$



4.7-6.

a)

Bas Var	Eq No	Z	Coefficient of						Right Side
			X1	X2	X3	X4	X5	X6	
Z	0	1	-2	2	-3	0	0	0	0
(0) X4	1	0	-1	1	1	1	0	0	4
X5	2	0	2	-1	1	0	1	0	2
X6	3	0	1	1	3	0	0	1	12

(1)

Bas Var	Eq No	Z	Coefficient of						Right Side
			X1	X2	X3	X4	X5	X6	
Z	0	1	4	-1	0	0	3	0	6
X4	1	0	-3	2	0	1	-1	0	2
X3	2	0	2	-1	1	0	1	0	2
X6	3	0	-5	4	0	0	-3	1	6

(2)

Bas Var	Eq No	Z	Coefficient of						Right Side
			X1	X2	X3	X4	X5	X6	
Z	0	1	2.5	0	0	0.5	2.5	0	7
X2	1	0	-1.5	1	0	0.5	-0.5	0	1
X3	2	0	0.5	0	1	0.5	0.5	0	3
X6	3	0	1	0	0	-2	-1	1	2

Optimal: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$, $z^* = 7$

b) The shadow prices are:

$$y_1^* = \frac{1}{2}, y_2^* = \frac{5}{2}, y_3^* = 0$$

They are the marginal values of resources 1, 2 and 3, respectively.

c)

Variables	2	-2	3	7	Optimal Value
	0	1	3		
Constraints	-1	1	1	4 <=	4
	2	-1	1	2 <=	2
	1	1	3	10 <=	12

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$3	Variables	0	-2.5	2	2.5	1E+30
\$C\$3	Variables	1	0	-2	1.6666667	1
\$D\$3	Variables	3	0	3	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$5	Constraints	4	0.5	4	1	2
\$F\$6		2	2.5	2	2	6
\$F\$7		10	0	12	1E+30	2