hwk3, 1 (5 points). Consider the linear programming problem

Minimize $z = x_1 + x_2$

subject to
$$-x_1 + x_2 \le 1$$
$$3x_1 + x_2 \le 13$$
$$x_1 + 3x_2 \ge 7$$
$$x_1 \ge 0,\ x_2 \ge 0.$$

(a) Sketch the feasible region.

(b) Sketch a few level curve of the objective function.

(c) Find the optimal solution by the graphical method.

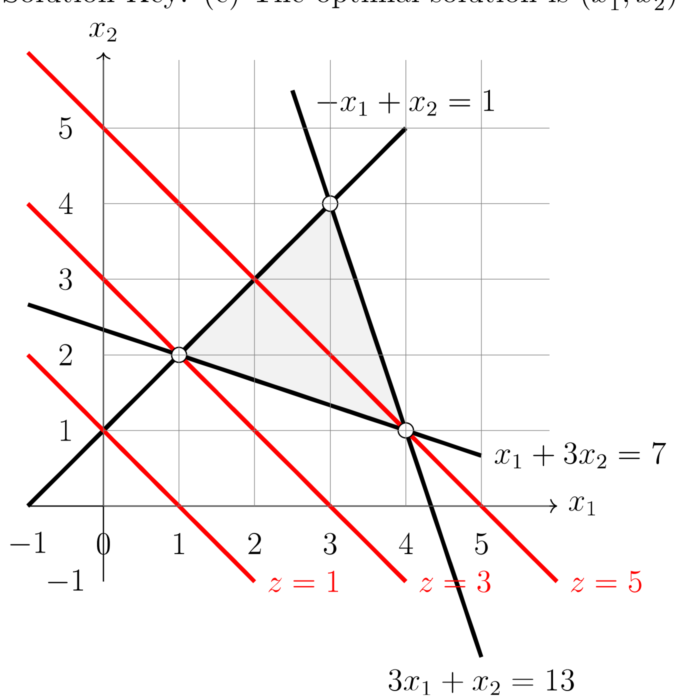
hwk3, 2 (5 points). Consider the linear programming problem

Maximize $z = 2x_1 - 2x_2 + 3x_3$

subject to
$$-x_1 + x_2 + x_3 \le 4$$
$$2x_1 - x_2 + x_3 \le 2$$
$$x_1 + x_2 + 3x_3 \le 12$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$$

Work through the simplex method step by step to solve the problem. Include details on optimality test, ratio test, entering variable, and leaving variable from one feasible echelon form to the next. State at each step the basic feasible solution. Intermediate tableaus between feasible echelon forms are not necessary to include, which can be substituted in place of brief descriptions on elementary row operations, such as 2R1+R0 → R0, 2R2, etc.

#1 Solution Key: (c) The optimal solution is $(x_1^*, x_2^*) = (1, 2)$, $z^* = 3$.



#2 Solution Key:

Tableau 0

		z	x_1	x_2	x_3	x_4	x_5	x_6	rhs	Ratio Test
	z	1	-2	2	-3	0	0	0	0	
basic variable	x_4	0	-1	1	1	1	0	0	4	4/1=4
	x_5	0	2	-1	1	0	1	0	2	2/1=2
	x_6	0	1	1	3	0	0	1	12	12/3=4

Feasible echelon form 0. Feasible Corner Solution: $(z, x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 0, 4, 2, 12)$.

Not optimal for having negative coefficients for R0 equation. Ratio Test: x_3 entering, x_5 leaving. EROs: 3R2+R0, -R2+R1, -3R2+R3.

Tableau 1

		z	x_1	x_2	x_3	x_4	x_5	x_6	rhs	Ratio Test
	z	1	4	-1	0	0	3	0	6	
basic variable	x_4	0	-3	2	0	1	-1	0	2	2/2=1
	x_3	0	2	-1	1	0	1	0	2	NA
	x_6	0	-5	4	0	0	-3	1	6	6/4

Feasible echelon form 1. Feasible Corner Solution: $(z, x_1, x_2, x_3, x_4, x_5, x_6) = (6, 0, 0, 2, 2, 0, 6)$.

Not optimal for having negative coefficients for R0 equation. Ratio Test: x_2 entering, x_4 leaving. EROs: 1/2R1, R1+R0, R1+R2, -4R1+R3.

Tableau 2/ Last Tableau

		z	x_1	x_2	x_3	x_4	x_5	x_6	rhs	Ratio Test
	z	1	5/2	0	0	1/2	5/2	0	7	
basic variable	x_2	0	-3/2	1	0	1/2	-1/2	0	1	
	x_3	0	1/2	0	1	1/2	1/2	0	3	
	x_6	0	1	0	0	-2	-1	1	2	

Optimal Test satisfied, final feasible echelon form. Optimal Solution: $(z^*, x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*) = (7, 0, 1, 3, 0, 0, 2)$.

hwk2, 1 (5 points). Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 7 & 3 \\ 2 & 4 & 6 & 4 \end{bmatrix}$. Consider the augmented matrix

$$B = [A \vdots I]$$

where I is the 3-by-3 identity matrix.

(a) Use elementary row operations to obtain the reduced row echelon form of B. At each step, record the corresponding elementary matrix E_i.

(b) Find the product of the elementary matrixes: $E = E_k \cdot E_{k-1} \cdots E_1$.

(c) Explain why E can be found from (a). Explain why E is the inverse matrix for $C = [a_1, a_2, a_4]$ where a_i is the ith column of A.

hwk2, 2 (5 points). The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit		
Department	Special Risk	Mortgage	Work-Hours Available
Underwriting	3	2	2400
Administration	0.5	1	800
Claims	2	0.2	1200

Formulate a linear programming model for this problem.

#1 Solution Key: #1(a)

$$B = [A \vdots I] = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 0 & 0 \\ 1 & 3 & 7 & 3 & 0 & 1 & 0 \\ 2 & 4 & 6 & 4 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[E_1=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}]{\text{-R1+R2}} \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 2 & -1 & 1 & 0 \\ 2 & 4 & 6 & 4 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[E_2=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}]{\text{-2R1+R3}} \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 1 \end{bmatrix} \xrightarrow[E_3=\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}]{\text{-2R2+R1}} \begin{bmatrix} 1 & 0 & -5 & -3 & 3 & -2 & 0 \\ 0 & 1 & 4 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[E_4=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}]{\text{1/2)R3}} \begin{bmatrix} 1 & 0 & -5 & -3 & 3 & -2 & 0 \\ 0 & 1 & 4 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1/2 \end{bmatrix} \xrightarrow[E_5=\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}]{\text{-R3+R1}} \begin{bmatrix} 1 & 0 & -5 & 0 & 0 & -2 & 3/2 \\ 0 & 1 & 4 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1/2 \end{bmatrix}$$

$$\xrightarrow[E_6=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}]{\text{-2R3+R2}} \begin{bmatrix} 1 & 0 & -5 & 0 & 0 & -2 & 3/2 \\ 0 & 1 & 4 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1/2 \end{bmatrix} = \text{rref}(B)$$

#1(b) $E = E_6 \cdot E_4 \cdots E_1 = \begin{bmatrix} 0 & -2 & 3/2 \\ 1 & 1 & -1 \\ -1 & 0 & 1/2 \end{bmatrix}$

#1(c) By one row operation on B, it is to multiply its elementary matrix E_i to the left of B, column by column, or blocks of columns by blocks of columns. So

$$E_6 \cdot E_5 \cdots E_1 \cdot B = [E \cdot A \vdots E \cdot I] = [e_1 \ e_2 \ \bar{a}_3 \ e_3 \vdots E] = \text{rref}(B),$$

showing the right-most 3-by-3 block of rref(B) is E.

Also, by the block matrix multiplication above, we see that

$$E \cdot C = [Ea_1 \ Ea_2 \ Ea_4] = [e_1 \ e_2 \ e_3] = I$$

showing $C^{-1} = E$.

#2 Solution Key: Let x_1 be the number of units in sales quota for special risk insurance and x_2 be the number of units in sales quota for mortgages. Then the LP problem is

Maximize $z = 5x_1 + 2x_2$
subject to $3x_1 + 2x_2 \leq 2400$
$0.5x_1 + x_2 \leq 800$
$2x_1 + 0.2x_2 \leq 1200$
$x_1 \geq 0, \ x_2 \geq 0.$
