Auto Assembly Analysis
Operations Research
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Abstract

Rachel Rosecrantz, the manager of the assembly plant of Automobile Alliance, must make decisions in several different situations regarding the number of Family Thrillseeker and Classy Cruiser car models that must be assembled to maximize the company's profit. In this report we apply linear programming using Microsoft Excel to each of these situations to aid in Rachel's decisions.

1 Introduction

Automobile Alliance is a large automobile manufacturing company. Rachel Rosencrantz is the manager of Alliance's assembly plant outside Detroit, MI, where two models from the family of midsized and luxury cars are assembled: the Family Thrillseeker and the Classy Cruiser. The Family Thrillseeker is a four-door sedan with vinyl seats, plastic interior, standard interior, and excellent gas mileage. It is marketed as a smart buy for middle-class families on a tight budget and generates a profit of $3600. The Classy Cruiser is a two-door luxury sedan with leather seats, wooden interior, custom features, and navigational capabilities. It is marketed toward upper-middle class families and generates a profit of $5400.

Rachel must decide how many Family Thrillseekers and Classy Cruisers to assemble to maximize the company's profit. The plant has a capacity of 48,000 labor-hours per month. It takes 6 labor-hours to assemble one Family Thrillseeker and 10.5 labor-hours to assemble one Classy Cruiser. The parts are shipped from other plants in the Michigan area to Rachel's assembly plant. Rachel will only be able to obtain 20,000 doors (10,000 left and 10,000 right) from the door supplier, due to a labor strike, and both the Family Thrillseeker and the Classy Cruiser use the same door part. A company forecast suggests that the demand for the Classy Cruiser is limited to 3500 cars, and there is no limit on the demand of the Family Thrillseeker. We investigate each of the following cases: [1]

1. We determine, based on the original information, how many Family Thrillseekers and Classy Cruisers to assemble.
2. The marketing department can pursue a targeted $500,000 advertising campaign that will raise the demand for the Classy Cruiser next month by 20 percent. We determine whether or not the advertising campaign should be undertaken.
3. Rachel can increase next month's plant capacity by using overtime labor; she can increase the plant's labor-hour capacity by 25 percent. We determine then how many of each model of car should be assembled.
4. We determine, based on Part 3, the maximum amount Rachel should be willing to pay for all overtime labor beyond the cost of this labor at regular time rates.
5. Rachel could use both the targeted advertising campaign and the overtime labor-hours. The campaign raises the demand of the Classy Cruiser 20 percent and the overtime labor-hours increase the plant's labor-hour capacity by 25 percent. We determine the number of each model of car to assemble in this case.

[1] Each of the following cases is independent except where otherwise indicated.

6. The advertising campaign costs \$500,000 and the overtime labor-hours cost \$1,600,000 beyond regular time rates. We determine whether or not the actions in Part 5 should be taken.

7. The company has determined that dealerships are heavily discounting the price of the Family Thrillseekers to move them off the lot. Because of a profit-sharing agreement with its dealers, the company is now making a profit of only \$2800 on the Family Thrillseeker. Given this new discounted price we determine the number of Family Thrillseekers and Classy Cruisers to assemble.

8. The company discovered quality problems with the Family Thrillseeker through random testing. There are problems with two of the four doors of a Family Thrillseeker sealing properly. Because of the now added quality testing at the end of the line for each Family Thrillseeker, it now takes 7.5 labor-hours to assemble a Family Thrillseeker. We determine the number of each model of car to assemble in this case.

9. The board of directors of Automobile Alliance wants to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classy Cruisers. They ask Rachel to determine by how much the profit of her assembly plant would decrease as compared to the profit found in Part 1. They then ask her to meet the full demand for Classy Cruisers if the decrease in profit is not more than \$2,000,000. We determine whether or not Rachel should meet the full demand for Classy Cruisers.

10. Finally, Rachel considers the simultaneous situations described in Parts 6,7, and 8. We determine whether or not she should undertake the advertising campaign and whether or not she should use overtime labor. We then determine the number of each model to be assembled in her final decision.

2 Analysis and Results

We first assign variables. Let x_1 denote the number of Family Thrillseekers to be assembled and let x_2 denote the number of Classy Cruisers to be assembled. Our basic profit equation is $Z = 3600x_1 + 5400x_2$. We now analyze and set up a linear programming model for each of the respective situations.

1. In this basic case we maximize $Z = 3600x_1 + 5400x_2$ subject to

$$
\begin{aligned}
&6x_1 + 10.5x_2 \leq 48000 \\
&4x_1 + 2x_2 \leq 20000 \\
&x_2 \leq 3500.
\end{aligned}
$$

   The first contraint represents the labor-hour capacity, the second constraint represents the maximum number of doors Rachel has available, and the third constraint represents the demand of the Classy Cruiser. In this case Rachel should have 3800 Family Thrillseekers and 2400 Classy Cruisers assembled with a maximum profit of \$26,640,000.

2. Since the advertising campaign costs \$500,000 we adjust our profit equation to be $Z = 3600x_1 + 5400x_2 - 500000$. Since the advertising campaign will raise the demand for the Classy Cruiser 20 percent, the demand is now 4200. Thus our constraints are

$$
\begin{aligned}
&6x_1 + 10.5x_2 \le 48000\\
&4x_1 + 2x_2 \le 20000\\
&x_2 \le 4200.
\end{aligned}
$$

In this case the maximum profit is \$26,140,000. This is less than the profit in Part 1 and so the company should not undertake the advertising campaign alone.

3. Since the labor-hours are increased by 25 percent, Rachel's plant now has a 60,000 labor-hour plant capacity. Thus we maximize $Z = 3600x_1 + 5400x_2$ subject to the constraints

$$
\begin{aligned}
&6x_1 + 10.5x_2 \le 60000\\
&4x_1 + 2x_2 \le 20000\\
&x_2 \le 3500.
\end{aligned}
$$

   In this case Rachel should have 3250 Family Thrillseekers and 3500 Classy Cruisers assembled for a maximum profit of \$30,600,000.

4. With overtime the company earns a profit of \$30,600,000 but without overtime the company earns a profit of \$26,640,000. Since overtime comes with a price, Rachel should be willing to pay at most the difference in these two profits for overtime costs, i.e. at most \$3,960,000.

5. Neglecting the costs of implementation, if Rachel uses both the advertising campaign and the overtime labor-hours, we maximize $Z = 3600x_1 + 5400x_2$ subject to the constraints

$$
\begin{aligned}
&6x_1 + 10.5x_2 \le 60000\\
&4x_1 + 2x_2 \le 20000\\
&x_2 \le 4200.
\end{aligned}
$$

   In this case 3000 Family Thrillseekers and 4000 Classy Cruisers should be assembled for a maximum profit of \$32,400,000.

6. We must now subtract the costs of the advertising campaign (\$500,000) and overtime labor (\$1,600,000) from the profit in Part 5. This results in a profit of \$30,300,000. If Rachel uses neither the campaign nor overtime labor, the profit will be \$26,640,000. If Rachel uses just the campaign, the profit will be \$26,140,000. If Rachel uses just overtime labor, the profit will be \$29,000,000. Thus Rachel should choose to use the advertising campaign and the overtime labor.

7. Since each new Thrillseeker profit is now \$2800, we change our profit equation to $Z = 2800x_1 + 5400x_2$. We keep the same constraints as in Part 1. In this case Rachel should have 1875 Family Thrillseekers and 3500 Classy Cruisers assembled for a maximum profit of \$24,150,000.

8. Since it now takes 7.5 labor-hours to assemble a Family Thrillseeker, we maximize $Z = 3600x_1 + 5400x_2$ subject to the constraints

$$\begin{aligned} 7.5x_1 + 10.5x_2 &\leq 48000 \\ 4x_1 + 2x_2 &\leq 20000 \\ x_2 &\leq 3500. \end{aligned}$$

   In this case Rachel should have 1500 Family Thrillseekers and 3500 Classy Cruisers assembled for a maximum profit of $24,300,000.

9. Since Rachel must analyze what happens when the demand for the Classy Cruiser is met, we maximize $Z = 3600x_1 + 5400x_2$ subject to the constraints

$$\begin{aligned} 6x_1 + 10.5x_2 &\leq 48000 \\ 4x_1 + 2x_2 &\leq 20000 \\ x_2 &= 3500. \end{aligned}$$

   In this case 1875 Family Thrillseekers and 3500 (of course) Classy Cruisers should be assembled for a maximum profit of $25,650,000. Now, the profit found in Part 1 was $26,640,000 and when we subtract this new profit we get a difference of $990,000. This difference is less than $2,000,000 and so Rachel should meet the full demand of the Classy Cruisers.

10. Rachel now considers whether or not to use advertising and/or overtime labor, given that the dealerships are heavily discounting the price of the Family Thrillseekers to move them off the lot and that quality problems have been discovered. Thus the company is making only a $2800 profit on each Family Thrillseeker, and it takes 7.5 labor-hours to assemble a Family Thrillseeker. Thus we have four options to consider.

    (a) Without advertising or using overtime labor we maximize $Z = 2800x_1 + 5400x_2$ subject to the constraints

$$\begin{aligned} 7.5x_1 + 10.5x_2 &\leq 48000 \\ 4x_1 + 2x_2 &\leq 20000 \\ x_2 &\leq 3500. \end{aligned}$$

    In this case Rachel should have 1500 Family Thrillseekers and 3500 Classy Cruisers assembled for a maximum profit of $23,100,000.

    (b) Without advertising but using overtime labor we maximize $Z = 2800x_1 + 5400x_2 - 1600000$ subject to the constraints

$$
\begin{aligned}
&7.5x_1 + 10.5x_2 \le 60000 \\
&4x_1 + 2x_2 \le 20000 \\
&x_2 \le 3500.
\end{aligned}
$$

In this case Rachel should have 3100 Family Thrillseekers and 3500 Classy Cruisers assembled for a maximum profit of \$25,980,000.

(c) With advertising but without using overtime labor we maximize $Z = 2800x_1 + 5400x_2 - 500000$ subject to the constraints

$$
\begin{aligned}
&7.5x_1 + 10.5x_2 \le 48000 \\
&4x_1 + 2x_2 \le 20000 \\
&x_2 \le 4200.
\end{aligned}
$$

In this case Rachel should have 520 Family Thrillseekers and 4200 Classy Cruisers assembled for a maximum profit of \$23,636,000.

(d) Advertising and using overtime labor we maximize $Z = 2800x_1 + 5400x_2 - 500000 - 1600000$ subject to the constraints

$$
\begin{aligned}
&7.5x_1 + 10.5x_2 \le 60000 \\
&4x_1 + 2x_2 \le 20000 \\
&x_2 \le 4200.
\end{aligned}
$$

In this case Rachel should have 2120 Family Thrillseekers and 4200 Classy Cruisers assembled for a maximum profit of \$26,516,000.

To summarize, we have the following table:

	Advertise	Don't Advertise
Overtime	26,516,000	25,980,000
No Overtime	24,136,000	23,100,000

Thus we conclude that Rachel should both advertise and use overtime in this case and should have 2120 Family Thrillseekers and 4200 Classy Cruisers assembled for a maximum profit of \$26,516,000.