Name:______________________________ Score:________

1(25pts) Mayoral candidate Jamie has \$40,000 left in the last week of campaign to buy TV, radio, or newspaper commercials. An estimate has been made by his aids and consultants about the additional votes he can won depending on the types of commercial run. These estimates are given in thousands of votes and in tens of thousand dollars as follows.

	Commercials		
Expenditures	**Newspaper**	**Radio**	**TV**
0	0	0	0
1	3	7	4
2	5	8	9
3	9	9	12
4	15	10	13

Use dynamic programming method to decide how much candidate Jamie needs to spend on these three types of medias in order to get the maximum additional votes

2(20pts) A jewelry store stocks up its 1-carat diamond inventory monthly according to this policy: It places an order of 3 diamonds if its 1-carat diamond inventory is empty by the last week of a month; 2 diamonds if it has 1 left; and no order otherwise. The order will be delivered in time for next month's business cycle. It is estimated that the monthly demand D_t for its 1-carat diamonds are independent, identically distributed according to the following table

n	0	1	2	≥ 3
$P\{D_t = n\}$	0.2	0.45	0.25	0.1

(a) Let X_t denote the number of 1-carat diamonds at the time of inventorying. What are the possible states of X_t?

(b) Explain why the stochastic process $\{X_t\}$ is Markovian.

(c) Construct the (1-step) transition matrix P.

(d) If $X_0 = 3$, what is the probability that the inventory would be empty in 2 months from now, i.e. $P\{X_2 = 0|X_0 = 3\}$?

3(20pts) A pizza restaurant can handle a maximum of 4 take-outs at any given time. Customers are turned away if there are 4 orders already placed. Orders are placed at a mean rate of 3 per hour according to a Poisson process. The restaurant can complete on average 5 orders per hour which is also exponentially distributed.

(a) Set up a rate diagram for this queueing system.

(b) Find C_n.

(c) Find the steady-state probabilities.

(d) What is the expected time in minute that a customer has to wait to pick up his/her order?

4(25pts) A single server queueing system employs a nonpreemptive discipline for its customers who arrive at a mean rate of 5 per hour according to a Poison process. A customer pays a fee of \$10 to get into the system. If a customer pays \$50 up front, he/she is moved to the nonpreemptive priority class over those who don't. It is estimated that about 10% of the customers are willing to pay for this queue priority. The service completion time is exponentially distributed with a mean completion time of 10 minutes.

(a) Find the expected the waiting times W_1, W_2 with W_1 for the priority class.

(b) If your time is worth \$50 per hour outside the queueing system and completely worthless while waiting, are you willing to pay for the priority status?

5(10pts) For an s-server queueing system, show that

$$L = \sum_{n=0}^{s-1} nP_n + L_q + s\left(1 - \sum_{n=0}^{s-1} P_n\right)$$

by using the definition of L, L_q in terms of the P_n.

End

Solution Key Math 428 Exam II Spring '02

1 (25pts). x_1, x_2, x_3 — expenditures in \$10,000 unit for Newspapers, Radio, TV ads.

s_1, s_2, s_3 — available amount for (x_1, x_2, x_3), (x_2, x_3), x_3.

s_3	$f_3^*(s_3)$	x_3^*
0	0	0
1	4	1
2	9	2
3	12	3
4	13	4

$f(s_2, x_2) = V_{x_2} + f_3^*(s_2 - x_2)$

$s_2 \backslash x_2$	0	1	2	3	4	f_2^*	x_2^*
0	0	–	–	–	–		
1	4	7	–	–	–	7	1
2	9	11	8	–	–	11	1
3	12	16	12	9	–	16	1
4	13	19	17	13	10	19	1

$s_1 \backslash x_1$	0	1	2	3	4	f_1^*	x_1^*
4	19	19	16	16	15	19	0, 1

opt.

x_1^*	x_2^*	x_3^*	$f_1^*(4)$
0	1	3	19
1	1	2	19

2 (20pts)
$$X_{t+1} = \begin{cases} \max\{3 - D_{t+1}, 0\}, & X_t = 0 \\ \max\{2 - D_{t+1}, 0\}, & X_t = 1 \\ \max\{X_t - D_{t+1}, 0\}, & X_t \geq 2 \end{cases} \Rightarrow$$

(a) $X_t \in \{0, 1, 2, 3\}$,

(b) X_{t+1} dep. only on X_t.

(c) $P_{00} = P\{D_{t+1} \geq 3\} = .1$, $P_{01} = P\{D_{t+1} = 2\} = .25$, $P_{02} = P\{D_{t+1} = 1\} = 0.45$, $P_{03} = P\{D_{t+1} = 0\} = 0.2$

$P_{10} =$ " $= .1$, $P_{11} =$ " $= .25$, $P_{12} =$ " $= 0.45$, $P_{13} =$ " $= 0.2$

$P_{20} = P\{D_{t+1} \geq 2\} = 0.35$, $P_{21} = P\{D_{t+1} = 1\} = .45$, $P_{22} = 0.2$, $P_{23} = 0$

$P_{30} = P\{D_{t+1} \geq 3\} = 0.1$, $P_{31} = 0.35$, $P_{32} = .45$, $P_{33} = 0.2$

$$P = \begin{bmatrix} .1 & .25 & .45 & .2 \\ .1 & .25 & .45 & .2 \\ .35 & .45 & .2 & 0 \\ .1 & .25 & .45 & .2 \end{bmatrix}$$

(d) $P_{30}^{(2)} = $ 4th row $\cdot$ 1 col. $= .1(.1) + .25(.1) + .45(.35) + .2(.1)$
$= .2125$

3 (20pts) (a) $0 \underset{5}{\overset{3}{\rightleftarrows}} 1 \underset{5}{\overset{3}{\rightleftarrows}} 2 \underset{5}{\overset{3}{\rightleftarrows}} 3 \underset{5}{\overset{3}{\rightleftarrows}} 4$

(b) $C_0 = 1$, $C_i = (\frac{3}{5})^i$ $i = 0, 1, 2, 3, 4$, $C_i = 0$, $i \geq 5$.

(c) $P_0 = 1/[1 + (\frac{3}{5}) + \cdots + (\frac{3}{5})^4] = \frac{1 - 3/5}{1 - (3/5)^5} = 0.43370$, $P_1 = 0.26022$, $P_2 = .156132$, $P_3 = .0936792$, $P_4 = 0.0562075$

(d) $L = 0P_0 + P_1 + 2P_2 + 3P_3 + 4P_3 = 1.0783$, $W = \frac{L}{\lambda} = .3595\,hr \doteq 22$ min.

4 (25pts). $\lambda_1 = .5$, $\lambda_2 = 4.9$, $\mu = 6$. $a_1 = \frac{\lambda_1}{\mu^2} = 0.0139$, $a_2 = a_1 + \frac{\lambda_2}{36} = 0.1389$

$b_0 = 1$, $b_1 = 1 - \frac{\lambda_1}{\mu} = 0.9167$, $b_2 = 0.1667$. $\Rightarrow W_1 = \frac{a_1}{b_0 b_1} + \frac{1}{\mu} = 0.1818\,hr \doteq 11$ min.

$W_2 = \frac{a_2}{b_1 b_2} + \frac{1}{\mu} = \cancel{\quad} 1.075\,hr \doteq 64.5$ min.

(b) WC — waiting cost. F — fixed fee.

class	WC	F	total cost
1	$50W_1 = 9.0909$	50	59.0909
2	$50W_2 = 53.7879$	10	63.7879

← minimum ⇒ pay \$50 to save time

5 (10pts) $L = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{s-1} nP_n + \sum_{n=s}^{\infty} nP_n = \sum_{n=0}^{s-1} + \sum_{n=s}^{\infty}(n-s)P_n + s\sum_{n=s}^{\infty} P_n = \sum_{n=0}^{s-1} + L_q + s(1 - \sum_{n=0}^{s-1} P_n)$