1 (30pt) One international airport has two runways, one used exclusively for takeoffs and the other exclusively for landing. Airplanes arrive according to a Poission process at a mean rate of 10 per hour. The time required for an airplane to land after receiving clearance for landing has an exponential distribution with a mean of 3 minutes. This process must be completed before giving clearance for landing to another plane. Airplanes awaiting clearance must circle the airport.
(a) Find the average number of airplanes waiting to receive clearance for landing.
(b) Find the fraction of time the number of airplanes circling the airport to receive landing clearance does not exceed 3.
(c) Find the probability of an airplane which needs to circle the airport for landing clearance for at least 30 minutes.

2 (35pts) You run a small company that depends on internet sales. You have 3 computer servers to manage the internet activity. These servers are very important. Each one breaks down about once a week with an exponentially distributed inter-breakdown time. About 80% of the time you can fix the problem quickly with an average repair time of ½ hour. The remaining 20% of the time, the repair is more difficult and the average repair time is 6 hours. Both repair times are exponentially distributed.
(a) Set up the rate diagram for this queue.
(b) Give the balance equations.
(c) Find the fraction of time all servers are working.

3 (35pts) A jewelry store stocks up its 1-carat diamond inventory monthly according to this policy: It places an order of 3 diamonds if its 1-carat diamond inventory is empty by the last week of a month; 2 diamonds if it has 1 left; and no order otherwise. The order will be delivered in time for next month's business cycle. It is estimated that the monthly demand D_t for its 1-carat diamonds can be modeled by a Poisson distribution with the rate parameter 2 per month.
(a) Let X_t denote the number of 1-carat diamonds at the time of inventorying. What are the possible states of X_t?
(b) Construct the transition matrix for the Markov chain.
(c) If X_0 = 3, what is the probability that the inventory would be empty in 2 months from now, i.e. P{X_2 = 0|X_0 = 3}?
(d) Find the mean first passage time that the store needs to order 3 diamonds to that the store does not need to order any.