PROBLEMS

The symbols to the left of some of the problems (or their parts) have the following meaning:

D: The demonstration example listed above may be helpful.
I: You may find it helpful to use the corresponding procedure in IOR Tutorial (the printout records your work).
C: Use the computer to solve the problem by applying the simplex method. The available software options for doing this include the Excel Solver or Premium Solver (Sec. 3.5), MPL/CPLEX (Sec. 3.6), LINGO (Supplements 1 and 2 to this chapter on the book's website and Appendix 4.1), and LINDO (Appendix 4.1), but follow any instructions given by your instructor regarding the option to use.

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

3.1-1. Read the referenced article that fully describes the OR study summarized in the application vignette presented in Sec. 3.1. Briefly describe how linear programming was applied in this study. Then list the various financial and nonfinancial benefits that resulted from this study.

D **3.1-2.*** For each of the following constraints, draw a separate graph to show the nonnegative solutions that satisfy this constraint.
(a) $x_1 + 3x_2 \le 6$
(b) $4x_1 + 3x_2 \le 12$
(c) $4x_1 + x_2 \le 8$
(d) Now combine these constraints into a single graph to show the feasible region for the entire set of functional constraints plus nonnegativity constraints.

D **3.1-3.** Consider the following objective function for a linear programming model:

Maximize $Z = 2x_1 + 3x_2$

(a) Draw a graph that shows the corresponding objective function lines for $Z = 6$, $Z = 12$, and $Z = 18$.
(b) Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis.

3.1-4. Consider the following equation of a line:

$$60x_1 + 40x_2 = 600$$

(a) Find the slope-intercept form of this equation.
(b) Use this form to identify the slope and the intercept with the x_2 axis for this line.
(c) Use the information from part (*b*) to draw a graph of this line.

D,I **3.1-5.*** Use the graphical method to solve the problem:

Maximize $Z = 2x_1 + x_2$,

subject to

$$\begin{aligned} x_2 &\le 10 \\ 2x_1 + 5x_2 &\le 60 \\ x_1 + x_2 &\le 18 \\ 3x_1 + x_2 &\le 44 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

D,I **3.1-6.** Use the graphical method to solve the problem:

Maximize $Z = 10x_1 + 20x_2$,

subject to

$$\begin{aligned} -x_1 + 2x_2 &\le 15 \\ x_1 + x_2 &\le 12 \\ 5x_1 + 3x_2 &\le 45 \end{aligned}$$

and

$$x_1 \ge 0, \; x_2 \ge 0.$$

3.1-7. The Whitt Window Company is a company with only three employees which makes two different kinds of hand-crafted windows: a wood-framed and an aluminum-framed window. They earn \$180 profit for each wood-framed window and \$90 profit for each aluminum-framed window. Doug makes the wood frames, and can make 6 per day. Linda makes the aluminum frames, and can make 4 per day. Bob forms and cuts the glass, and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass and each aluminum-framed window uses 8 square feet of glass.

The company wishes to determine how many windows of each type to produce per day to maximize total profit.
(a) Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Sec. 3.1. Then construct and fill in a table like Table 3.1 for this problem, identifying both the activities and the resources.
(b) Formulate a linear programming model for this problem.
D,I **(c)** Use the graphical method to solve this model.
I **(d)** A new competitor in town has started making wood-framed windows as well. This may force the company to lower the price they charge and so lower the profit made for each wood-framed window. How would the optimal solution change (if at all) if the profit per wood-framed window decreases from \$180 to \$120? From \$180 to 60? (You may find it helpful to use the Graphical Analysis and Sensitivity Analysis procedure in IOR Tutorial.)
I **(e)** Doug is considering lowering his working hours, which would decrease the number of wood frames he makes per day. How would the optimal solution change if he makes only 5 wood frames per day? (You may find it helpful to use the Graphical Analysis and Sensitivity Analysis procedure in IOR Tutorial.)

3.1-8. The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical subject to

$$-x_1 + x_2 \le 2$$
$$x_2 \le 3$$
$$kx_1 + x_2 \le 2k + 3, \quad \text{where } k \ge 0$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

The solution currently being used is $x_1 = 2$, $x_2 = 3$. Use graphical analysis to determine the values of k such that this solution actually is optimal.

D **3.1-14.** Consider the following problem, where the values of c_1 and c_2 have not yet been ascertained.

$$\text{Maximize} \quad Z = c_1x_1 + c_2x_2,$$

subject to

$$2x_1 + x_2 \le 11$$
$$-x_1 + 2x_2 \le 2$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1 and c_2. (*Hint:* Separate the cases where $c_2 = 0$, $c_2 > 0$, and $c_2 < 0$. For the latter two cases, focus on the ratio of c_1 to c_2.)

3.2-1. The following table summarizes the key facts about two products, A and B, and the resources, Q, R, and S, required to produce them.

Resource	Resource Usage per Unit Produced: Product A	Product B	Amount of Resource Available
Q	2	1	2
R	1	2	2
S	3	3	4
Profit per unit	3	2	

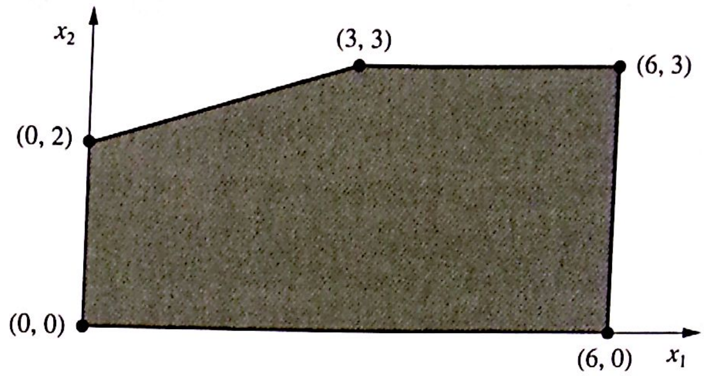
All the assumptions of linear programming hold.

(a) Formulate a linear programming model for this problem.

D,I **(b)** Solve this model graphically.

(c) Verify the exact value of your optimal solution from part (*b*) by solving algebraically for the simultaneous solution of the relevant two equations.

3.2-2. The shaded area in the following graph represents the feasible region of a linear programming problem whose objective function is to be maximized.



Label each of the following statements as True or False, and then justify your answer based on the graphical method. In each case, give an example of an objective function that illustrates your answer.

(a) If (3, 3) produces a larger value of the objective function than (0, 2) and (6, 3), then (3, 3) must be an optimal solution.

(b) If (3, 3) is an optimal solution and multiple optimal solutions exist, then either (0, 2) or (6, 3) must also be an optimal solution.

(c) The point (0, 0) cannot be an optimal solution.

3.2-3.* This is your lucky day. You have just won a \$10,000 prize. You are setting aside \$4,000 for taxes and partying expenses, but you have decided to invest the other \$6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any *fraction* of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

(a) Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Sec. 3.1. Then construct and fill in a table like Table 3.1 for this problem, identifying both the activities and the resources.

(b) Formulate a linear programming model for this problem.

D,I **(c)** Use the graphical method to solve this model. What is your total estimated profit?

$$2x_1 + 3x_2 = 12$$
$$2x_1 + x_2 \geq 8$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

D **3.4-7.** Consider the following problem, where the value of c_1 has not yet been ascertained.

Maximize $\quad Z = c_1x_1 + 2x_2,$

subject to

$$4x_1 + x_2 \leq 12$$
$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1.

D,I **3.4-8.** Consider the following model:

Minimize $\quad Z = 40x_1 + 50x_2,$

subject to

$$2x_1 + 3x_2 \geq 30$$
$$x_1 + x_2 \geq 12$$
$$2x_1 + x_2 \geq 20$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Use the graphical method to solve this model.
(b) How does the optimal solution change if the objective function is changed to $Z = 40x_1 + 70x_2$? (You may find it helpful to use the Graphical Analysis and Sensitivity Analysis procedure in IOR Tutorial.)
(c) How does the optimal solution change if the third functional constraint is changed to $2x_1 + x_2 \geq 15$? (You may find it helpful to use the Graphical Analysis and Sensitivity Analysis procedure in IOR Tutorial.)

3.4-9. Ralph Edmund loves steaks and potatoes. Therefore, he has decided to go on a steady diet of only these two foods (plus some liquids and vitamin supplements) for all his meals. Ralph realizes that this isn't the healthiest diet, so he wants to make sure that he eats the right quantities of the two foods to satisfy some key nutritional requirements. He has obtained the nutritional and cost information shown at the top of the next column.

Ralph wishes to determine the number of daily servings (may be fractional) of steak and potatoes that will meet these requirements at a minimum cost.
(a) Formulate a linear programming model for this problem.
D,I (b) Use the graphical method to solve this model.
C (c) Use a computer to solve this model by the simplex method.

Ingredient	Grams of Ingredient per Serving		Daily Requirement (Grams)
	Steak	Potatoes	
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	$4	$2	

3.4-10. Web Mercantile sells many household products through an online catalog. The company needs substantial warehouse space for storing its goods. Plans now are being made for leasing warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease expire) at least once but not every month.

The space requirement and the leasing costs for the various leasing periods are as follows:

Month	Required Space (Sq. Ft.)
1	30,000
2	20,000
3	40,000
4	10,000
5	50,000

Leasing Period (Months)	Cost per Sq. Ft. Leased
1	$ 65
2	$100
3	$135
4	$160
5	$190

The objective is to minimize the total leasing cost for meeting the space requirements.
(a) Formulate a linear programming model for this problem.
C (b) Solve this model by the simplex method.

3.4-11. Larry Edison is the director of the Computer Center for Buckly College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants are required:

Time of Day	Minimum Number of Consultants Required to Be on Duty
8 A.M.–noon	4
Noon–4 P.M.	8
4 P.M.–8 P.M.	10
8 P.M.–midnight	6