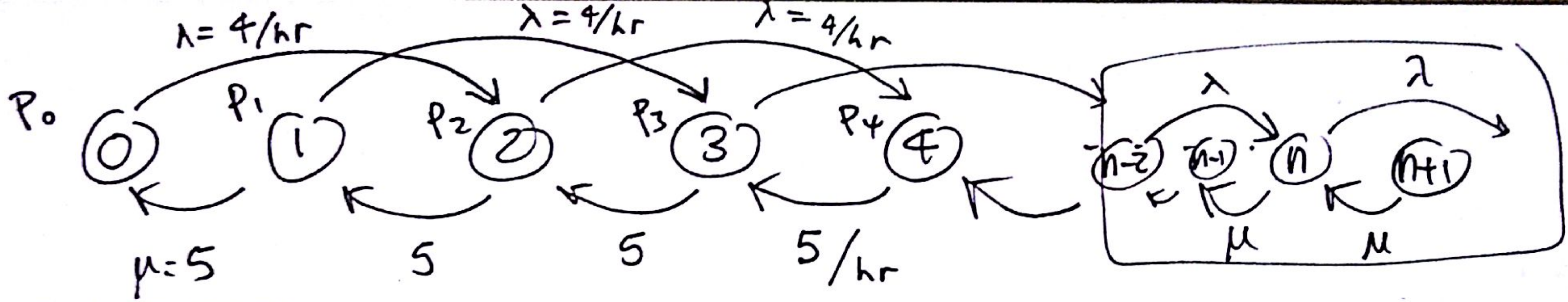
①




$$\boxed{\text{mean in-rate at state } n \quad = \quad \text{mean out-rate at state } n.}$$

$$\mu P_1 = \lambda P_0$$

$$\mu P_2 = (\mu + \lambda) P_1$$

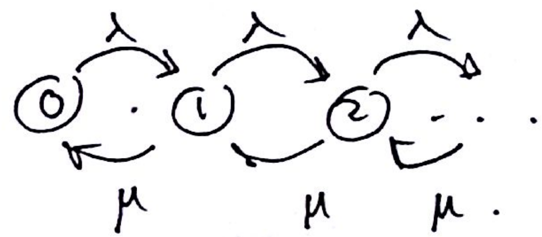
$$\lambda P_0 + \mu P_3 = (\mu + \lambda) P_2$$

$$\vdots$$

state n: $\lambda P_{n-2} + \mu P_{n+1} = (\mu + \lambda) P_n.$

$$\vdots$$

(c)



§17.7 ⓒ

- M/G/1 model.

M — exp. dist. input process w/ mean rate $\lambda$.

G — general dist. for service time T with

$E(T) = \frac{1}{\mu}$, and $var(T) = \sigma^2$, $\rho = \frac{\lambda}{\mu} < 1$

$(G = M, \; var(T) = \frac{1}{\mu^2})$.

| | G | M/D/1 w/ $\sigma = 0$. | M/$E_k$/1. |
|---|---|---|---|
| ✓ $P_0$ | $1-\rho$ | $1-\rho$ | " |
| $L_q$ | $\boxed{\frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)}}$ | $\boxed{\frac{\rho^2}{2(1-\rho)}}$ | $\boxed{\frac{\lambda^2/k\mu^2 + \rho^2}{2(1-\rho)}}$. |
| ✓ $L$ | $L_q + \rho$ | $L_q + \rho$ | |
| Little's. $W_q$ | $\frac{L_q}{\lambda}$ | $\frac{L_q}{\lambda}$. | |
| $W$ | $\frac{L}{\lambda} = W_q + \frac{1}{\mu}$ | $\frac{L}{\lambda} = W_q + \frac{1}{\mu}$. | |

- M/D/$ model.

D — degenerate dist.. special case of G-model.

M/D/1: $E(T) = \frac{1}{\mu}$ , $var(T) = \sigma^2 = 0$, $\rho = \frac{\lambda}{\mu} < 1$.

Rmk: For M/M/1 model, $L_q = \frac{\rho^2}{1-\rho}$

$$L_q(M/D/1) = \frac{\rho^2}{2(1-\rho)} = \frac{1}{2} L_q(M/M/1).$$

$k=1$
$\Rightarrow E_k = E_1 = M$.

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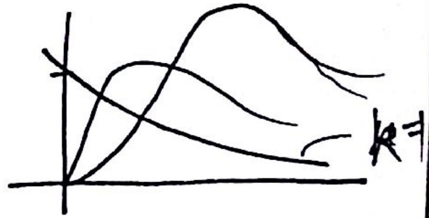
M/$E_k$/s model.

$E_k$ — Erlang dist. w/ prob. density function.

$$f_T(t) = \begin{cases} \frac{(\mu k)^k}{(k-1)!} t^{k-1} e^{-k\mu t} & , t \geq 0 \\ 0 & , t \leq 0. \end{cases}$$



w/ parameter k — integer, shape parameter

and $\mu$ — mean ~~couple~~ out-put rate.

$\int_0^\infty f_T(t)\,dt = 1$.

$E(T) = \frac{1}{\mu}$ , $var(T) = \frac{1}{k}\frac{1}{\mu^2}$.

**Property** : $T_1, \cdots, T_K$ are $k$ indep. random variables w/ identical exp. dist. w/ mean rate $\mu_0$

Then $T = T_1 + \cdots + T_K$ is of Erlang dist. w/ parameter $K$ and $\mu := \frac{\mu_0}{K}$

17.7 #5.

(a) ✓

(b) 

model $M/E_2/1$

Repair time $T = T_1 + T_2$ is Erlang dist. w/ $k=2$, $\mu = \mu_0/2 = 4/2 = 2$.

(c) $E(T) = \frac{1}{\mu} = \frac{1}{2}$, $\sigma^2 = var(T) = \frac{1}{K\mu^2} = \frac{1}{2\cdot 2^2} = 0.125$, $\rho = \frac{\lambda}{\mu} = \frac{1}{2}$

$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} = 0.375$, $L = L_q + \rho = 0.875$.

(d) $W = \frac{L}{\lambda} = 0.875$ hr $= 52.5$ min.