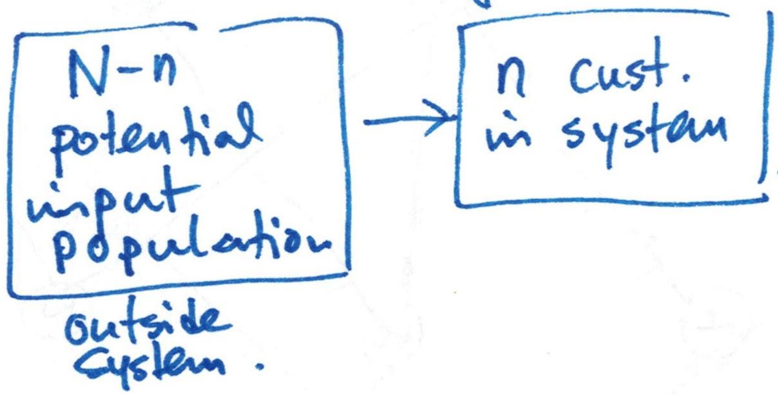
- Finite Calling Population N.

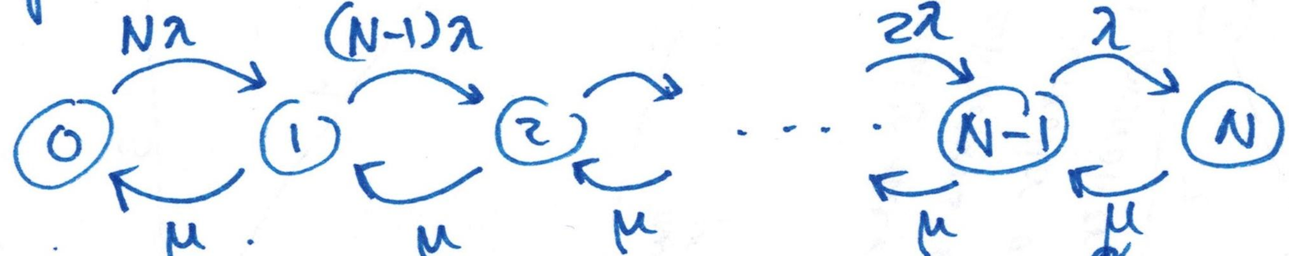


Outside time T has exp. dist. w/ const. λ

$$\Rightarrow \quad \lambda_n = \begin{cases} (N-n)\lambda, & n \le N \\ 0, & n \ge N. \end{cases}$$

w/ ~~[illegible]~~ $s = 1$.

Rate Diagram:



$$C_0 = 1, \quad C_n = \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1} = \frac{N(N-1)\cdots(N-n+1)\lambda^n}{\mu^n} = \begin{cases} \frac{N!}{(N-n)!}\rho^n, & n \\ 0 & n > \end{cases}$$

$$L = \sum_{n=0} n P_n = \sum_{n=0} n \rho^n (1-\rho) = (1-\rho)(0\cdot 1 + 1\cdot\rho + 2\rho^2 + 3\rho^3 + \cdots$$

$$= (1-\rho)\rho\left[1 + 2\rho + 3\rho^2 + 4\rho^3 + \cdots + n\rho^{n-1} + \cdots\right].$$

$$= (1-\rho)\rho\left[1' + (\rho)' + (\rho^2)' + (\rho^3)' + \cdots + (\rho^n)' + \cdots\right]$$

$$= (1-\rho)\rho\left[\underline{1 + \rho + \rho^2 + \rho^3 + \cdots + \rho^n + \cdots}\right]'$$

$$= (1-\rho)\rho\cdot\left(\frac{1}{1-\rho}\right)' = (1-\rho)\rho\,\frac{1}{(1-\rho)^2} = \frac{\rho}{1-\rho}.$$

$$L_q = \frac{\rho^2}{1-\rho}. \quad \text{by } L = L_q + \rho \Rightarrow L_q = L - \rho = \frac{\rho}{1-\rho} - \cdots$$
$$= \frac{\rho^2}{1-\rho}.$$

$$W = \frac{L}{\lambda} = \frac{1}{\lambda}\frac{\rho}{1-\rho}$$

$$W_q = \frac{L_q}{\lambda} = \frac{1}{\lambda}\frac{\rho^2}{1-\rho}.$$

$\mathcal{W}$ — random variable for time in the system.

$\mathcal{W}_q$ — " " " " " " queue.

<u>Then</u>: $P(\mathcal{W} > t) = e^{-\mu(1-\rho)t}$ — exp. dist.

$P(\mathcal{W}_q > t) = \rho e^{-\mu(1-\rho)t}$

See textbook for M/M/s w/ s>1 model.

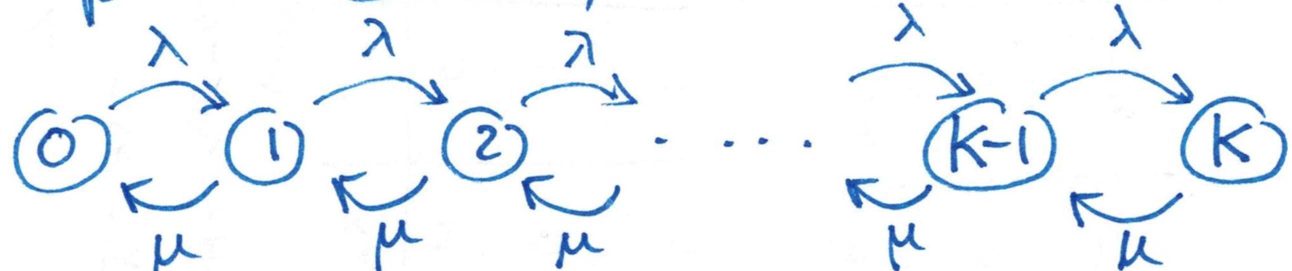
More Example :

- Finite Queue, variation of M/M/s model (M/M/s/K)

K — ~~Que~~ max. ~~queue~~ system length, to which the queue length is K−s

$$\lambda_n = \begin{cases} \lambda & \text{for } n = 0, 1, 2, \cdots, K-1 \\ 0 & \text{for } n \geq K \end{cases}$$

Rate Diagram for M/M/1/K :



$c_0 = 1,\ c_1 = \frac{\lambda}{\mu},\ c_2 = \frac{\lambda^2}{\mu^2},\ \cdots,\ c_{K-1} = \rho^{K-1},\ c_K = \frac{\lambda_{K-1}\lambda_{K-2}\cdots\lambda_0}{\mu_K\,\mu_{K-1}\cdots\mu}$

$c_{K+1} = \frac{\lambda_K \cdots \lambda_0}{\mu_{K+1}\cdots\mu_1} = 0,$

$$C = c_0 + c_1 + \cdots + c_K = 1 + \rho + \rho^2 + \cdots + \rho^K = \begin{cases} \frac{1-\rho^{K+1}}{1-\rho}, & \rho \neq 1 \\ K+1, & \rho = 1 \end{cases}$$

$$P_0 = \frac{1}{C} = \frac{1-\rho}{1-\rho^{K+1}}$$

or $\frac{1-\rho^{K+1}}{1-\rho} = K+1$ for

$$p_n = C_n p_0 = \begin{cases} \dfrac{\rho^{n}(1-\rho)}{1-\rho^{K+1}}, & n \le K \\ 0, & n \ge K+1 \end{cases}$$

$$L = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} = \sum_{n=0}^{K} n p_n = \sum_{n=0}^{K} n \rho^n \frac{(1-\rho)}{1-\rho^{K+1}} = \frac{\rho(1-\rho)}{1-\rho^{K+1}} \sum_{n=1}^{K} n \rho^{n}$$

$$= \frac{\rho(1-\rho)}{1-\rho^{K+1}} \left[\sum_{n=0}^{K} \rho^n \right]' = \frac{\rho(1-\rho)}{1-\rho^{K+1}} \left(\frac{1-\rho^{K+1}}{1-\rho} \right)'$$

$$= \frac{\rho\cancel{(1-\rho)}}{(1-\rho^{K+1})} \frac{-(K+1)\rho^{K}(1-\rho) + (1-\rho^{K+1})}{(1-\rho)^{\cancel{2}}}$$

$$= \frac{-(K+1)\rho^{K+1}}{1-\rho^{K+1}} + \frac{\rho}{1-\rho}.$$

$S=1$

$$\Rightarrow L_q = L - \rho = \frac{\rho^2}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}}$$

$$\bar{\lambda} = \sum_{n=0}^{K} \lambda_n p_n = \lambda. \Rightarrow W, \quad W_q, \quad \cdots$$