$$E(X(t)) = \sum_{n=0}^{\infty} n P\{X(t) = n\} = \sum_{n=0}^{\infty} n \cdot \frac{(\alpha t)^n e^{-\alpha t}}{n!} = \sum_{n=1}^{\infty} \frac{(\alpha t)^n e^{-\alpha t}}{(n-1)!}$$

$$= e^{-\alpha t} (\alpha t) \sum_{n=1}^{\infty} \frac{(\alpha t)^{n-1}}{(n-1)!} \underset{k=n-1}{=} e^{-\alpha t} (\alpha t) \sum_{k=0}^{\infty} \frac{(\alpha t)^k}{k!} = e^{-\alpha t} \cdot (\alpha t) e^{\alpha t}$$

$$= \alpha t$$

Average rate of arrival:

$$\frac{E(X(t))}{t} = \alpha \longleftarrow \text{arrival or completion rate.}$$

- ~~Independent~~ Aggregated Poisson process with parameter $\alpha_1, \alpha_2, \cdots, \alpha_n$ is again a Poisson process.

$$X(t) = X_1(t) + X_2(t) + \cdots + X_n(t)$$

Then $X(t)$ is of Poisson distribution with parameter

$$\alpha = \alpha_1 + \alpha_2 + \cdots + \alpha_n.$$

- Conversely, if an aggregated input process is Poisson with α arrival rate and each independent type of customers arrivals with probability p_i, $i = 1, 2, \cdots, n$, $\sum p_i = 1$.

Then the input process of customers of type i can be modeled as Poisson with arrival rate $\lambda_i = \lambda p_i$.

Ex:

$P\{T < 1\} = 1 - e^{-\alpha(1)} = 1 - e^{-\frac{1}{2}}$

$= .393$

(ii) $P\{1 < T < 2\}$

$\cancel{= P(T < 2) - P(T \leq 1)}$

$\cancel{=}$

$P\{1 < T < 2 \mid T > 1\}$

$= \dfrac{P\{1 < T < 2,\ T > 1\}}{P(T > 1)}$

$= \dfrac{P\{1 < T < 2\}}{P(T > 1)} = \dfrac{P(T < 2) - P(T < 1)}{P(T > 1)}$

$= \dfrac{1 - e^{-\alpha(2)} - (1 - e^{-\alpha 1})}{e^{-\alpha(1)}} = \dfrac{\cancel{1} - e^{-2\alpha} + e^{-\alpha}}{e^{-\alpha}}$

$= 1 - e^{-\alpha} = .393.$

λ — mean arrival rate.

$\bar{T}$ — " interarrival time.

$\lambda = \frac{1}{\bar{T}} \iff \bar{T} = \frac{1}{\lambda}$.

$\bar{T} = 2hr.$

μ — mean service rate

$\mu = \frac{1}{2}$ cust/hr.

c (ii): $P\{8(1)=1\}_{n=1} = \dfrac{(\alpha(1))^1 e^{-\alpha(1)}}{1!} = \dfrac{\frac{1}{2}e^{-\frac{1}{2}}}{1.}$

$= .303$

$P\{8(1)=0\} = \dfrac{(\alpha(1))^0 e^{-\alpha(1)}}{0!} = e^{-\frac{1}{2}} = .607.$

$P\{8(1) \geq 2\} = 1 - P\{8(1)=0\} - P\{8(1)=1\}$

$= 1 - (.607) - (.303) = 1 - .91 = 0.09$

$\mu = \frac{1}{2}$

$2\mu = 1/hr$

$P\{T > 1\}$ w/ $\alpha = 2\mu$.

$P(8(1) = 0)$

$P(8(1) = 1)$

(a) ii $P\{\ldots\} = 1 - e^{-\alpha(1)}$

$= .393$