$$\Rightarrow \quad W = \frac{L}{\lambda} = \frac{4}{8} = \frac{1}{2}\ hr = 30\ min.$$

$$W_q = \frac{L_q}{\lambda} = \frac{2}{8} = \frac{1}{4}\ hr = 15\ min.$$

$W - W_q = 30 - 15 = 15$ min. — average visiting time to see the doctor.

$$\mu = \frac{1}{W - W_q} = \frac{1}{.25} = 4\ pat./hr.$$

~~Util~~ Utilization factor $\rho = \frac{\lambda}{s\mu} = \frac{8\ patients/hr}{2\ (4\ patients/hr)} = 1.$

$\Rightarrow$ The doctors are always busy.

What if $\boxed{s = 3}$:

$$L_q = 0 \cdot P_0 + 0\, P_1 + 0 \cdot P_2 + 0 \cdot P_3 + 1\, P_4 + 2P_5 + 3P_6 + 4P_7 + 5P_8$$

$$= \_\_\_$$

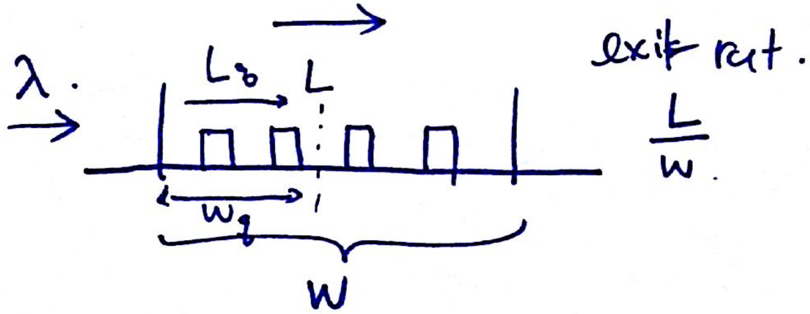
$$W_q = \frac{L_q}{\lambda}, \quad \mu, \quad \rho.$$

For any queueing system, we six system parameters and three formulas.

- L — system length. $L = \sum_{n=0}^{\infty} n P_n$
- L_q — queue length. $L_q = \sum_{n=s}^{\infty} (n-s) P_n$, s — # of server.
- W — time in the system.
- W_q — time in the queue.
- λ — arrival rate
- μ — service rate for each busy server.

<u>Thm</u>: Little's Formula.

- $\frac{L}{W} = \lambda$
- $\frac{L_q}{W_q} = \lambda$
- $\mu = \frac{1}{W - W_q}$



Rmk: The system is determined by any ~~of the~~ three of the six parameters

$$\#6.\quad \rho = \frac{\lambda}{s\mu} \overset{s=1}{=\!=} \frac{\lambda}{\mu} = \frac{\lambda}{\frac{1}{W - W_q}} = \lambda(W - W_q) = \lambda W - \lambda W_q$$ ⑧

$$= L - L_q = (0\cdot p_0 + 1\cdot p_1 + 2p_2 + \cdots) - (0\cdot p_0 + 0 p_1 + 1\cdot p_2 + 2\cdot p_3 + \cdots)$$

$$= 0\cdot p_0 + \underbrace{p_1 + p_2 + p_3 + \cdots + p_0}_{} - p_0 = 1 - p_0 .$$

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§17.4. Exponential Distribution.

One way to model the event (random) of arrival.

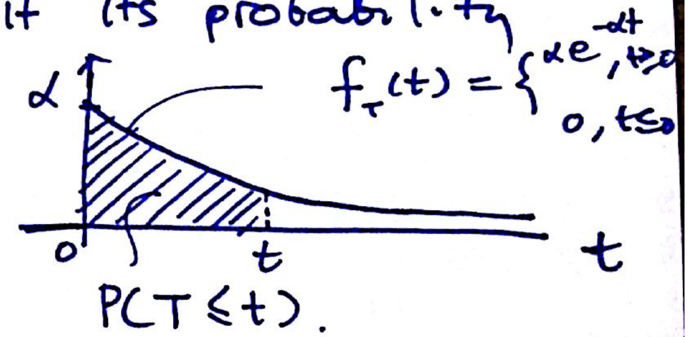
T — a random variable representing <u>interarrival time</u>

~~For~~ <u>Average (mean) interarrival time</u>, $\bar{T}$, $\Rightarrow \bar{\lambda} = \frac{1}{\bar{T}}$ — average arrival rate.

Need a model for the probability distribution of T.

<u>**Def**</u>: The random variable T is said to have an <u>exponential</u> distribution with parameter α if its probability ~~dis~~ density function is

$$f_T(t) = \begin{cases} \alpha e^{-\alpha t} & \text{for } t \ge 0 \\ 0 & \text{for } t < 0 . \end{cases}$$



a

Then

$$P(T \le t) = \int_0^t f_T(s)\,ds$$

$$= \int_0^t \alpha e^{-\alpha s}\,ds = 1 - e^{-\alpha t}$$

$$P(T < \infty) = 1 \quad \left(\lim_{t\to\infty} P(T \le t) = \lim_{t\to\infty}\left(1 - e^{-\alpha t}\right)\right).$$

$$E(T) = \int_0^\infty t\, f_T(t)\,dt = \int_0^\infty \alpha t e^{-\alpha t}\,dt$$

$$= \lim_{t\to\infty}\left(-se^{-\alpha s} - \tfrac{1}{\alpha}e^{-\alpha s}\Big|_0^t\right) = \frac{1}{\alpha}.$$

$$\Rightarrow \alpha = \frac{1}{E(T)}$$ — ~~arri~~ arrival rate.

1(b) cont'd: iff $x > \frac{1}{2}$,