Ex: Find the optimal strategies for P1, P2 for this game

P1 \ P2	1	2	3	min
1	-3	-1	5	-3
2	2	0	1	0
3	4	-2	-4	-4
max	4	0	5	
min		0		

payoff matrix for P1.

max min for P1 = $\underline{v} = 0$ (V of P1)

minmax for P2 = $\bar{v} = 0$ (V_2 of P2)

Goal is to both players to maximize their returns or to minimize their lost.

Minmax Method for problem solving

⇒ Solution: P1 plays his Strategy 2

P2 " her " 2

with $\underline{v} = 0 = \bar{v}$

This is a stable game.

Ex:

P1 \ P2	1	2	3	min	max
1	0	-2	2	-2	-2 = maxmin, $\underline{v}$
2	5	4	-3	-3	
3	2	3	-4	-4	
max	5	4	2		
min					

minmax

$\bar{v} = 2$.

Game value for each player is different.

$\bar{v} = 2 \neq \underline{v} = -2$.

This is not stable.

(No single strategy to play all the time for each player.

Repeatedly played Games.

Let $A = \begin{bmatrix} a_{11} \cdots a_{1n} \\ \vdots \\ a_{m1} \cdots a_{mn} \end{bmatrix}$ be payoff matrix to P1 w/

Strategies $s_1, s_2, \cdots, s_m$ for P1, and strategies $t_1, t_2, \cdots, t_n$ to P2 (on the columns).

~~Let N plays~~ The game is played N times, $N \gg 1$.

Let $n_{ij} \geq 0$ be the number of play with strategy i and strategy j used by P1 and P2 respectively.

Then total payoff for P1:

$$a_{11} n_{11} + a_{12} n_{12} + \cdots + a_{mn} n_{mn} = \sum_{i,j} a_{ij} n_{ij}$$

Average return / payoff per play:

$$\frac{\sum_{i,j} a_{ij} n_{ij}}{N} = \sum_{i,j} a_{ij} \frac{n_{ij}}{N}, \text{ with } \sum_{i,j} n_{ij} = N.$$

$$\text{or } \sum_{i,j} \frac{n_{ij}}{N} = 1.$$

$\frac{n_{ij}}{N}$ —— probability (p_{ij}) of ~~player~~ (i,j) strategy being played

~~i.e.~~ p_{ij}?

~~p~~

Let $x_i \geq 0$ be the probability of P1 playing his strategy i with $\sum_{i=1}^{m} x_i = 1$.

Let $y_j \geq 0$ be the " " P2 " her " j with $\sum_{j=1}^{n} y_j = 1$.

~~Asa~~ Assume independent strategy plays by the players.

By the independence assumption.

$$p_{ij} = x_i y_j$$

Def: the expect payoff value (per play) for player 1 is

$$E(x,y) = \sum_{i,j} a_{ij} x_i y_j, \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}, \ x_i \geq 0, \ \sum_i x_i = 1$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}, \ y_j \geq 0, \ \sum_j y_j = 1.$$

The goal of the game is ~~to~~ for player 1 to maximize $E(x,y)$ and for player 2 to maximize its return $-E(x,y)$ or is to minimize $E(x,y)$.