(A) Adapt to other forms of primal LP.

Big-M method.

Ex. max $z = x_1 + 2x_2$

sub. to $x_1 + x_2 \leq 10$

$2x_1 + x_2 \geq 14$

$x_1 \geq 0,\ x_2 \geq 0$.

Introduce slack variable x_3 $\longrightarrow$ and artificial variable x_4 with penalty. $-Mx_4$ so that the final opt. soln. must have $x_4 = 0$.

max $z = x_1 + 2x_2 - Mx_4$

sub to. $x_1 + x_2 + x_3 = 10$

$2x_1 + x_2 + x_4 = 14$

$\Leftrightarrow$ Solve

$$z - (1)x_1 - (2)x_2 - (-M)x_4 = 0$$
$$x_1 + x_2 + x_3 = 10$$
$$2x_1 + x_2 + x_4 = 14$$

w/ $x_i \geq 0$ and max of z.

$$\begin{array}{c} R_0 \\ \vec{R_1} \\ R_2 \end{array} \left[\begin{array}{ccccc|c} 1 & -(1) & -(2) & 0 & -(-M) & 0 \\ 0 & 1 & 1 & 1 & 0 & 10 \\ 0 & 2 & 1 & 0 & 1 & 14 \end{array}\right]$$

not in feas. echelon form

• Initialization: $-MR_2 + R_0 \rightarrow$ new R_0

$$\begin{array}{c|cccccc} B.V & z & x_1 & x_2 & x_3 & x_4 & \\ \hline z & 1 & -(1+2M) & -(2+M) & 0 & 0 & -14M \\ x_3 & 0 & 1 & 1 & 1 & 0 & 10 \\ x_4 \ (x_1) & 0 & 2 & 1 & 0 & 1 & 14 \end{array}$$

In F.E.F. w/ BFS: $(0,0,10,14)$, $z = -14M$.

• Then Use the standard simplex method.

2

Ex: max: $z = x_1 + 2x_2$

sub to. $x_1 + x_2 \le 10$

$2x_1 + x_2 \ge 14$

$x_1 \ge 0,\ x_2 \ge 0$.

- Add slack variable $x_3 \ge 0$
- Int. surplus variable $x_4 \ge 0$
- Add artificial variable $x_5 \ge 0$ w/ penalty $-Mx_5$.

$\longrightarrow$ max: $z = x_1 + 2x_2 - Mx_5$

sub. to, $x_1 + x_2 + x_3 = 10$

$2x_1 + x_2 - x_4 + x_5 = 14$

$\boxed{x_4 = 2x_1 + x_2 - 14 \ge 0}$

→ Initialization → simplex method.

Def: An auxiliar variable is either a slack var., or a surplus var., or an artificial var.

Ex. min $z = c^T x$ sub.to $\cancel{Ax \le b}$ $\cancel{x \ge 0}$ whatever const. $\Longleftrightarrow$ max $w = -c^T x$ sub.to $\cancel{Ax \le b}$ $\cancel{x \ge 0}$ whatever const.

Also. $a^T x \le b \iff -a^T x \ge -b$. $\Rightarrow$ w.l.o.g.

we can assume the rhs of the const is always ≥ 0.

• Shadow Price.

Look at the z equation through the step of simplex method by row operation.

Consider: Primal Standard LP.

$$\max.\ z = c^T x$$

$$\text{sub.}\quad Ax \le b,\quad A = A_{m\times n},\ x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n,\ c = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix},\ b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$x \ge 0, \text{ and w.l.o.g. } b \ge 0.$$

$$\Longleftrightarrow \quad \begin{aligned} z - c^T x &= 0 \\ Ax + I x_s &= b \iff [A \vdots I]\begin{bmatrix} x \\ x_s \end{bmatrix} = b,\quad x_s = \begin{bmatrix} x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix}. \\ x \ge 0,\ x_s &\ge 0, \end{aligned}$$

By elementary row operation, let y_i be the scalar which is multiplied to row i of $[A \vdots I]\begin{bmatrix} x \\ x_s \end{bmatrix} = b$ and then added to row 0. $A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ w/ $a_i = [a_{i1}, \cdots, a_{in}]$, let $y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$

The corresponding z equation then is

$$\boxed{z - c^T x + y^T A x + y^T x_s = y^T b}.$$