$\underline{\underline{Ex}}$: max: $z = 3x_1 + 5x_2$

sub. to
$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 \le 18$$
$$x_1 \ge 0,\ x_2 \ge 0.$$

$\Leftrightarrow$

Aug. LP

max. $z = 3x_1 + 5x_2$
$$x_1 + x_3 = 4$$
$$2x_2 + x_4 = 12$$
$$3x_1 + 2x_2 + x_5 = 18$$
$$x_i \ge 0$$

$$\Leftrightarrow \quad \begin{aligned} z - 3x_1 - 5x_2 + 0x_3 + 0x_4 + 0x_5 &= 0 \\ x_1 + x_3 &= 4 \\ 2x_2 + x_4 &= 12 \\ 3x_1 + 2x_2 + x_5 &= 18 \end{aligned}$$

w/ $x_i \ge 0$ — feasibility and

max of z — obje.

$\Leftrightarrow$ By simplex method (in matrix or tabular form). Assume $b \ge 0$.

$$\begin{array}{c|cccccc:c|c} \text{Basic variable} & z & x_1 & x_2 \text{ (Enter)} & x_3 & x_4 & x_5 & \text{rht} & \text{Ratio Test} \\ \hline z & 1 & -(+3) & -(+5) & 0 & 0 & 0 & 0 & \\ x_3 & 0 & 1 & 0 & 1 & 0 & 0 & 4 & \frac{4}{0} = \infty \\ x_4 \text{ (leaving)} & 0 & 0 & 2 & 0 & 1 & 0 & 12 & \frac{12}{2} = \boxed{6} \\ x_5 & 0 & 3 & 2 & 0 & 0 & 1 & 18 & \frac{18}{2} = 9 \end{array}$$

- Feasible echelon form.
  $\Rightarrow$ One BFP: $x = (0, 0, 4, 12, 18)$ w/ $z = 0$
- Optimal Test: If $z = z_0$ at BFP a local max. No since there are (positive coefficient) in ~~z row~~.
  x_2 — entering variable to be a basic variable for the next FEF (feas. Ech. Form)
  The posible new soln. value = smallest + value of b_i/a_{i2}

- From ratio test, $\min_i (b_i/a_{ij})$ w/ x_j entering), x_4 is leaving to become nonbasic, i.e. $x_4 = 0$ in the next step (FEF).
- The cross-over entry is to become a new leading one entry in the next step.

- Use row operation to ~~deriv~~ derive the next FEF.
  - Multiple row 3 by 1/2 to get new R_3
  - (-2) new $R_3 + R_4 \rightarrow$ new R_4.
  - $(+5) R_3 + R_1 \rightarrow$ new R_1

$\Rightarrow$ new FEF.

enter.

$$\begin{array}{c|c|c}
\text{b.v.} & \begin{matrix} z & x_1 & x_2 & x_3 & x_4 & x_5 & \text{rhs.} \end{matrix} & \text{Ratio Test} \\
\begin{matrix} z \\ x_3 \\ x_2 \\ x_5 \end{matrix} & \left[\begin{array}{cccccc|c} 1 & -(+3) & 0 & 0 & -(5/2) & 0 & 30 \\ 0 & 1 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 6 \\ 0 & 3 & 0 & 0 & -1 & 1 & 6 \end{array}\right] & \begin{matrix} \\ 4/1=4 \\ 6/0=\infty \\ \frac{6}{3}=\boxed{2} \end{matrix}
\end{array}$$

leave

- BFP: $x = (0, 6, 4, 0, 6)$ w/ $z = 30$
- Opt. Test. No ~~since~~ because of $-(+3)$ coefficient.

$\Downarrow$ $R_4/2 \rightarrow R_4$, $-R_4 + R_2 \rightarrow R_2$, old $R_4 + R_1 \rightarrow R_1$

$$\begin{array}{c}
\\ x_3 \\ x_2 \\ x_1
\end{array}
\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -(-3/2) & -(-1) & 36. \\ 0 & 0 & 0 & 0 & 1/3 & -1/3 & 2 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 6 \\ 0 & 1 & 0 & 0 & -1/3 & 1/3 & 2 \end{array}\right]$$

BFP: $x = (2, 6, 2, 0, 0)$

$z = 36$, max.

Opt. test. yes.