Consider standard LP in $x \in \mathbb{R}^n$: (1)

$$\begin{aligned} \text{max.} \quad & z = c^T x + z_0 \\ \text{sub. to} \quad & Ax \le b \qquad \text{where } A = [a_{ij}]_{m \times n} \\ & x \ge 0 \end{aligned}$$

Feasible Region: $D = \{x : Ax \le b,\ x \ge 0\}$ — n-dimensional polygone w/ corners (vertex).

<u>Fact</u>: · A corner of D is a basic feasible point or solution.

· Boundary of D are hyperplanes defined by $Ax = b$ or $x = 0$ for some componenents. and each basis feasibe point is the solution of exactly n equality equations for the hyperplane boundary.

<u>Def</u>: Consider $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \le b_i$. let $x_{n+i} \ge 0$ and $x_{n+i} = b_i - (a_{i1}x_1 + \cdots + a_{in}x_n) \ge 0$, i.e

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + x_{n+i} = b_i$$

x_{n+i} is called the ith <u>slack variable</u>.

Let $\mathbb{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ \boxed{\begin{matrix} x_{n+1} \\ \vdots \\ x_{n+m} \end{matrix}} \end{bmatrix} \in \mathbb{R}^{n+m}$

Let $\mathbb{C} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{n+m}$. $x_s = \begin{bmatrix} x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} \in \mathbb{R}^m$.

standard LP
$$\begin{aligned} z &= c^T x \\ Ax &\le b \\ x &\ge 0 \end{aligned} \quad \Longrightarrow \quad \boxed{\begin{aligned} z &= C^T \mathbb{X} \\ [A \vdots I]\,\mathbb{X} &= b \\ \mathbb{X} &\ge 0 \end{aligned}}$$

Augmented form,