Ex. A company makes two products, product 1 (doors), product 2 (windows) in batches, each brings \$3k and \$5K ~~per bat~~ in revenues per batch respectively. Each product can only be manufactured in one of three plants of the following capacities: (1)

- plant 1 can make product 1 only, at 1 hr per batch for a weekly maximum ~~hour~~ 4 hours.
- plant 2 can make product 2 only, at 2 hr/batch for 12 hr max per week.
- plant 3 can make both at 3 hr/batch for product 1, 2 hr/batch for product 2 ~~product 1 and 2 respectively~~ for 18 hr max per week.

Maximize the weekly revenue from the two products.

Solution: Define the decision variables

x_1 —— # of batches for product 1 per week, ≥ 0

x_2 —— # of .. .. .. 2 .. .. ≥ 0

Objective:

max. $z = 3x_1 + 5x_2$ in \$K per week.

Subject to:

$$1 \cdot x_1 \leq 4 \quad (\text{hr / week})$$

$$2x_2 \leq 12$$

- Any solution, or point of D, of the constraints $Ax \le b$, $x \ge 0$ is a feasible point.
- The corner points of the bdry of D ~~is~~ are the <u>basic feasible solutions or points</u>

BFS	nonfeasible points
(0,0)	(0,9) $z(9) > 12$
(4,0)	(4,6)
(0,6)	(6,0)
(2,6)	
(4,3).	

$\begin{cases} x_2 = 6 \\ 3x_1 + 2x_2 = 18 \end{cases} \Rightarrow (2,6)$

$\begin{cases} x_1 = 4 \\ 3x_1 + 2x_2 = 18 \end{cases} \Rightarrow (4,3)$

<u>Obs</u>: . BF solutions or points are solutions to n-equations in $\mathbb{R}^n$ w/ each eq. defines the bdry of D.

. But not all solutions to the bdry equations are feasible.

For $\begin{cases} x_1 = 0 \\ 3x_1 + 2x_2 = 18 \end{cases} \Rightarrow (0,9)$ — not feasible nonfeasible point.

- Sketch a few level curves of $z = 3x_1 + 5x_2$

Pick a few levels:

$z = 0$: $0 = 3x_1 + 5x_2$ $(0,0), (1, -\frac{3}{5})$

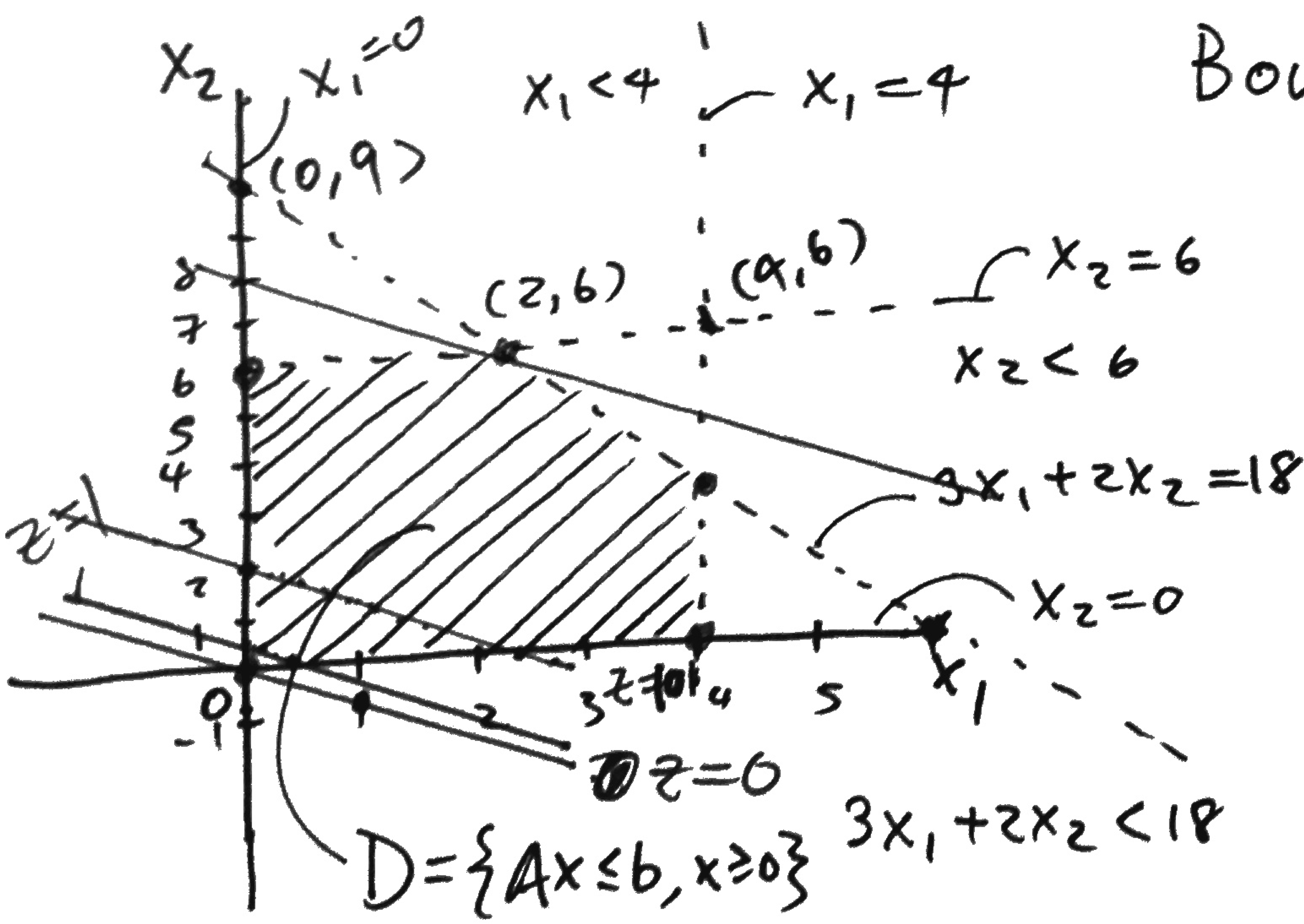
$z = 1$ $1 = 3x_1 + 5x_2$

$z = 10$ $10 = 3x_1 + 5x_2$, $(0,2)$

$$3x_1 + 2x_2 \le 18$$
$$x_1 \ge 0, \quad x_2 \ge 0$$

By graphical method.

- Sketch the feasible region: $D = \{ x = (x_1, x_2)^T : Ax \le b, x \ge 0 \}$ with $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$, in $\mathbb{R}^2$
- Sketch a few level curves $z = 3x_1 + 5x_2 = c^T x$ with $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$
- Solve the problem by ~~the~~ observation.



Boundaries of D:

$x_1 = 0, \quad x_2 = 0$

$x_1 = 4$

$2x_2 = 12 \iff x_2 = 6$

$3x_1 + 2x_2 = 18$

w/ $(0, 9) = (0, 9)$

$(6, 0)$

D is n-dimensional polygone in $\mathbb{R}^n$ w/ line segments as boundary and corner points, i.e. vertex.

- By method of looking, $x = (x_1, x_2) = (2, 6)$ is the optimal solution with the max. value at
  $z = 3(2) + 5(6) = \$36$ K / week

Rmk: **Thm:** For max $z = c^T x$ sub. to $Ax \le b$, $x \ge 0$, optimal solution must take place at one basic feasible solution, i.e. a corner point of the feasible region.

Thm: Any local max. point (solution) is necessarily a global max. solution to standard LP.

$$\boxed{\text{w/ Standard LP: } \quad \begin{aligned} \max: \ & z = c^T x \\ \text{sub. to. } \ & Ax \le b \\ & x \ge 0. \end{aligned}}$$

"pf". By convexity of $D = \{Ax \le b, x \ge 0\}$
i.e if $x_1, x_2 \in D$, then $x = \alpha x_1 + (1-\alpha) x_2 \in D$
for all $0 \le \alpha \le 1$