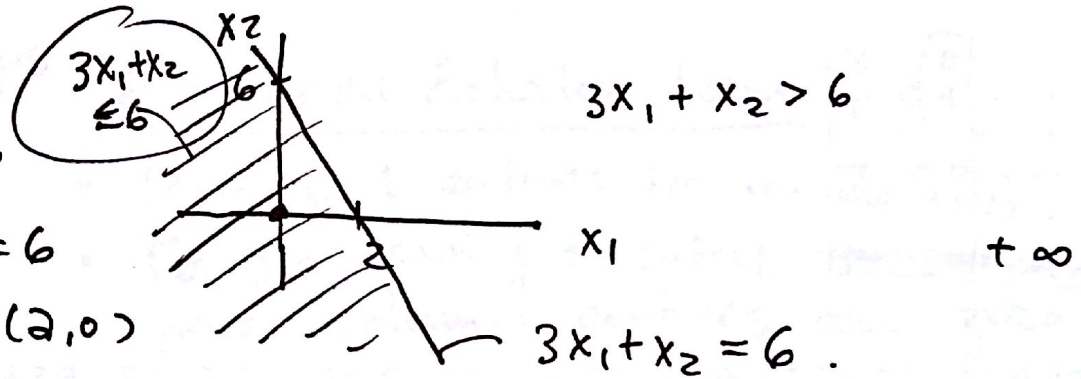
Review:

Region: $3x_1 + x_2 \leq 6$



Boundary: $3x_1 + x_2 = 6$

intercept: $(0,6)$, $(2,0)$

test at ∞: $x_1 \approx \infty$, $3x_1 + x_2 \sim +\infty > 6$.

at $(0,0)$: $3(0)+0 = 0 \leq 6$.

Solving system of linear equation.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m. \end{cases}$$

In matrix: $\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$, $A\vec{x} = \vec{b}$, $A = [a_{ij}]$.

$\Rightarrow [A \vdots \vec{b}]$

~~Rank(A)~~

Elementary row operations.

(1) Interchange two rows.

(2) Multiply a row by a nonzero constant.

(3) Add a constant multiple of one row to another row.

$[A|b]$ $\xrightarrow{\text{E. row operations}}$ & Reduced Echelon form $\begin{bmatrix} 1 & 0 & \cdots & * \\ 0 & 1 & \cdots & * \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & * \end{bmatrix}$

- leading 1 entries for nonzero rows.
- For the leading 1 entries all other column entries are zero.

$\Rightarrow$ Solution can be obtained easily.

Linear Programming / Optimation with linear functionals.

Objective function: maximize $z = f(x) = c_1x_1 + c_2x_2 + \cdots + c_nx_n + z_0$

with $x_1, x_2, \cdots, x_n$ decision variables

Subject to constraints:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m \\ x_1 \ge 0,\ x_2 \ge 0,\ \cdots,\ x_n &\ge 0 . \end{aligned} \quad (*)$$

(*) is called the standard form of LP.

Ex. Prototypical Example:

Ex

	Production Time (per batch in hr) 1 (door)	2 (windows)	Weekly capacity
plant 1	1	~~0~~	4
plant 2	—	2	12
plant 3	3	2	18
Revenue per batch.	$3k	$5k	

Decision variable in # of batches x_1 x_2.

Max. revenue: $Z = 3x_1 + 5x_2$

Sub. to:

$1 \cdot x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0, \; x_2 \geq 0$.

} in weekly set up.

$\Rightarrow$ A standard form of LP.

Ex: max. $z = x_1 + x_2$

sub. to. $x_1 + 2x_2 \leq 2$

$x_1 \geq 0, \; x_2 \geq 0$.

Graphical method for small problem.

Simplex method for general, large problems.