Math 314 Summer '11 Exam 1

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (20 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.
(Recall: an $m \times n$ matrix is one that has m rows and n columns.)

   (a) T or F? The column space col(A) is preserved by elementary row operations on A.

   (b) T or F? The null space null(A) is preserved by elementary row operations on A.

   (c) T or F? The span of a set of nonzero vectors $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is always a subspace.

   (d) T or F? For matrix multiplication, if $AB = 0$, then either $A = 0$ or $B = 0$.

   (e) T or F? A set of linearly independent vectors $\{\mathbf{v}_1, ..., \mathbf{v}_k\}$ forms a basis for its own span, span$(\{\mathbf{v}_1, ..., \mathbf{v}_k\})$.

   (f) T or F? If an $n \times n$ matrix A is invertible, the row vectors of A form a basis for R^n.

   (g) T or F? The rank(A) is equal to the rank(A^T).

   (h) T or F? Let A by an $m \times n$ matrix. Then rank(A) + nullity$(A) = m$.

   (i) T or F? Every system of three equations in two unknowns has exactly one solution.

   (j) T or F? If A is an $m \times n$ matrix, and $m < n$, then rank$(A) = m$.

2. (10 pts) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find $\mathbf{x}$ if you know that $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Solve the equation for $\mathbf{x}$: $AB\mathbf{x} - \mathbf{b} = 0$.

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3. (20 pts) (a) Find the inverse of the given matrix using the Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 1 & 0 \end{bmatrix}$$

(b) Write the inverse A^{-1} as a product of elementary matrixes.

(c) Write the matrix itself A as a product of elementary matrixes.

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4. (20 pts)

The following matrices are row-equivalent:

$$A = \begin{bmatrix} 2 & -1 & 3 & 1 & 8 \\ 1 & 3 & -9 & 4 & -2 \\ 0 & 1 & -3 & 1 & 3 \\ -2 & 3 & -9 & 1 & 7 \\ 3 & -1 & 3 & 2 & -1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 1 & -3 & 2 & 0 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer the following questions and **briefly** justify your reasoning:

(a) Find a basis for the row space, row(A), of A.

(b) Find a basis for the column space, col(A), of A.

(c) Find a basis for the null space, null(A), of A.

(d) Express the fourth column of A as a linear combination of the other columns.

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5. (10 pts) Write B as a linear combination of A_1, A_2, A_3 if possible.

$$B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}.$$

6. (10 pts) Let $\mathbf{u} = [4, 3, 2, 1]$ and $\mathbf{v} = [1, -1, 1, -1]$.

(a) Find the projection vector proj $_{\mathbf{u}}\mathbf{v}$.

(b) Find a vector $\mathbf{w}$ which is from the span$\{\mathbf{u}, \mathbf{v}\}$ and is perpendicular to $\mathbf{u}$.

(c) Write $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

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7. (5 pts) Prove that the rank(A) is equal to the rank(A^T).

8. (5 pts) If A is invertible, prove that its transpose A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$.

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2 Bonus Points: The state bird of Kansas is (a) The Northern Cardinal , (b) The Blue Jay, (c) The Western Meadowlark, (d) none of the above. (... *The End*)