Math 314 Spring '16 Exam 2

Name: ____________________ **Score:** ____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

   (a) T or F? The zero vector $\mathbf{0}$ is always an eigenvector of any matrix A.

   (b) T or F? If an $n \times n$ invertible matrix A is similar to a matrix B then the column space of B spans $\mathbb{R}^n$.

   (c) T or F? An invertible matrix A is always diagonalizable.

   (d) T or F? If λ and μ are eigenvalues of A and B, respectively, then $\lambda + \mu$ is an eigenvalue of $A + B$.

2. (15 pts) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 7 & 8 & 10 \\ 1 & 2 & 2 \end{bmatrix}$. You are given the fact that the transpose A^T is row equivalent to $B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   (a) Find a basis for the row space of A that are row vectors of A. Justify your answer.

   (b) Find a basis for the null space of the transpose A^T.

3. (20 pts) Let $\mathcal{B} = \{1 - t, 1 - t + t^2, t - t^2\}$ be a basis of $\mathbb{P}_2$ and $\mathcal{C} = \{1, t, t^2\}$ be its standard basis.

(a) Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, $P_{\mathcal{C} \leftarrow \mathcal{B}}$.

(b) Use (a) to find the change-of-coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$, $P_{\mathcal{B} \leftarrow \mathcal{C}}$. (Use calculator to save time.)

(c) Let $p = 1 + t + t^2$. Use (b) to find the coordinate of p with respect to $\mathcal{B}$, $[p]_{\mathcal{B}}$.

(d) Use (c) to write p as a linear combination of the polynomials in $\mathcal{B}$.

4. (15 pts) Let $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 0 & -1 \end{bmatrix}$.

(a) Find the characteristic equation for A.

(b) Find all eigenvalues of A.

(c) Verify that 1 is an eigenvalue, and find the eigenspace $E_{\lambda=1}$ of the eigenvalue 1.

5. (15 pts) The eigenvectors of matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ are $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find P and D to diagonalize A as $A = PDP^{-1}$.

6. (20 pts) (a) Let $M = \{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} : a_{ij}$ are arbitrary real numbers$\}$. Assume M is a vector space. Show that the set of matrices $H = \{\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} : a, b, c$ are arbitrary real numbers$\}$ is a subspace of M.

(b) Show that this map $T(A) = [1, 2]A$ with A from H is a linear transformation from H to the space of row vectors of $\mathbb{R}^2$.

(c) What is the standard basis for H?

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2 Bonus Points: The word *algebra* comes from (a) Latin, (b) Greek, (c) Arabic, or (d) Hindu.