Math 314 Spring 2014 Exam 2

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

   (a) T or F? λ is an eigenvalue of A if and only if null$(A - \lambda I)$ has a nonzero vector.

   (b) T or F? An invertible matrix A is always diagonalizable.

   (c) T or F? Zero is always an eigenvalue of non-invertible matrix.

   (d) T or F? If the determinant of a matrix is 1 then the rank of the matrix is also 1.

   (e) T or F? The zero subspace space $E = \{\mathbf{0}\}$ can be the eigenspace of a matrix.

   (f) T or F? An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.

2. (15 pts) Given the following similarity relation $P^{-1}AP = D$.

$$\begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \overbrace{\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 \end{bmatrix}}^{A} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) What are the eigenvalues of A and their multiplicities?

   (b) Find a basis for the eigenspace of the largest eigenvalue.

   (c) Is $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$ an engenvector of A? Justify your answer.

3. (15 pts) **Change-of-basis**. Consider the bases $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$.

(a) Verify that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is the coordinate $[\mathbf{x}]_{\mathcal{B}}$ in the basis $\mathcal{B}$ for the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(b) Find the change-of-basis matrix $\mathcal{P}_{\mathcal{C} \leftarrow \mathcal{B}}$.

(c) Find the coordinate $[\mathbf{x}]_{\mathcal{C}}$ in the basis $\mathcal{C}$ for $\mathbf{x}$.

4. (15 pts) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Find a basis consisting of vectors of the set for the space $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. (Show work to justify.)

5. (15 pts) Diagonalize matrix $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, i.e., find a diagonal matrix D and an invertible matrix P and P^{-1} so that $A = PDP^{-1}$. (You must show work to receive credit.)

6. (15 pts) Let $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$.

(a) Use a combination of row reduction, row and column expansion to find the characteristic equation of A. (No work no credit.)

(b) Is $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ an eigenvector of A? Justify your answer.

7. (10 pts) You are given the fact that $\lambda = -3$ is an eigenvalue of $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$. Find a basis for its eigenspace E_{-3}.

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2 Bonus Points: True or False: Lincoln has always been the capital of Nebraska. (*Page 4. ... The End*)