Math 314 Spring 2014 Exam 1

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

1. (10 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. (Recall: an $m \times n$ matrix is one that has m rows and n columns.)

   (a) T or F? Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Then it is onto if and only if it is one-to-one.

   (b) T or F? The span of a set of nonzero vectors $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is always a subspace.

   (c) T or F? For matrix multiplication, if $AB = 0$, then either $A = 0$ or $B = 0$.

   (d) T or F? The trivial solution $\mathbf{x} = 0$ is the only solution for every homogeneous equation $A\mathbf{x} = 0$.

2. (15 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotate a vector counterclockwise 45° and then project it to the x-axis.

   (a) Find its standard matrix $A = [T(\mathbf{e}_1), T(\mathbf{e}_2)]$. Show work.

   (b) Use the matrix to find the image $T(\mathbf{x})$ of the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

3. (25 pts) Let $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 4 & -1 & 7 \\ 1 & 2 & 0 & 5 \end{bmatrix}$ and it is given that $\text{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be the column vectors of the matrix A. Solve the following problems.

(a) Write each non-pivot column as a linear combination of the pivot columns.

(b) Can $\mathbf{v}_3$ be in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. If yes, write it as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$. If not, explain why.

(c) Can $\mathbf{v}_1$ be in $\text{Span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. If yes, write it as a linear combination of $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$. If not, explain why.

(d) Find a maximal linearly independent subset of the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

4. (15 pts) Set up a balance equation only for the following chemical reaction:

$$FeS_2 + O_2 \rightarrow Fe_2O_3 + SO_2$$

Do not solve for the solutions.

5. (15 pts) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$.

(a) Use elementary row reduction to find the inverse A^{-1}.

(b) Find the elementary matrixes for the row operations you used above to obtain the inverse.

(c) Write A^{-1} as a product of elementary matrixes.

(d) Write A as a product of elementary matrixes.

6. (20 pts) Let $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 4 & -1 & 7 \\ 1 & 2 & 0 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$. Find all solutions to $A\mathbf{x} = \mathbf{b}$. Express the solution as a linear combination of vectors.