Math 314 Fall '16 Exam 2

Print Your Name Legibly: ______________________ **Score:** ____________

Instructions: You must show supporting work to receive full and partial credits. No textbook, lecture notes, or formula sheets are allowed.

1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

   (a) T or F? The pivot columns of the reduced row echelon form of a matrix A form a basis for the column space of A.

   (b) T or F? The nonzero rows of the reduced row echelon form of a matrix A form a basis for the row space of A.

   (c) T or F? A matrix $A_{2\times 2}$ is always diagonalizable if it has two distinct eigenvalues.

   (d) T or F? If A and B are similar, then both have the same eigenvectors.

2. (15 pts) Let $A = \begin{bmatrix} 1 & 1 & 7 & 1 \\ 0 & 2 & 8 & 2 \\ 2 & 2 & 14 & 3 \end{bmatrix}$.

   (a) Find a basis for the column space of A

   (b) Find a basis for the null space of A.

3. (20 pts) Let $\mathcal{B} = \{1-t, t-t^2, 1+t^2\}$ be a basis of $\mathbb{P}_2$ and $\mathcal{C} = \{1, t, t^2\}$ be its standard basis.

(a) Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, $P_{\mathcal{C}\leftarrow\mathcal{B}}$.

(b) Use (a) to find the change-of-coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$, $P_{\mathcal{B}\leftarrow\mathcal{C}}$. (Use calculator to save time.)

(c) Let $p(t) = 1$. Use (b) to find the coordinate of p with respect to $\mathcal{B}$, $[p]_{\mathcal{B}}$.

(d) Use (c) to write p as a linear combination of the polynomials in $\mathcal{B}$.

4. (15 pts) Let $A = \begin{bmatrix} 3 & 2 \\ -4 & -1 \end{bmatrix}$.

(a) Find the characteristic equation for A.

(b) You are given the fact that $\lambda = 1 - 2i$ is a complex eigenvalue of A. Use this information to an complex-valued eigenvector of the eigenvalue.

5. (10 pts) You are given the fact that $\lambda = -3$ is an eigenvalue of $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$. Find a basis for its eigenspace $E_{\lambda=-3}$. (You must show work to receive credits.)

6. (15 pts) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Find an invertible matrix P so that $P^{-1}AP$ is diagonalized and find the diagonal matrix D as well.

7. (10 pts) (a) Let $M = \{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$: a_{ij} are arbitrary real numbers$\}$. Assume M is a vector space. Show that the set of matrices $H = \{\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$: a, b are arbitrary real numbers$\}$ is a subspace of M.

(b) What is the standard basis for H?

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2 Bonus Points: How many Electoral College votes does the State of Nebraska have for the presidential election? (a) 1, (b) 3, (c) 5, or (d) 7.