Math 314 Fall 2016 Exam 1

Print Your Name Legibly: ______________________ **Score:** ______________

Instructions: You must show supporting work to receive full and partial credits. No textbook, lecture notes, or formula sheets are allowed.

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1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. (Recall: an $m \times n$ matrix is one that has m rows and n columns.)

   (a) T or F? For matrixes A, B, C, if $AB = AC$ then $B = C$.

   (b) T or F? If a square matrix A is row-equivalent to the identity matrix, then it must be invertible.

   (c) T or F? Every transformation is a linear transformation.

   (d) T or F? If the columns of an $n \times n$ matrix A are linearly dependent, then they span the space $\mathbb{R}^n$.

   (e) T or F? Every system of two equations in three unknowns has infinitely many solutions.

2. (15 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflect a vector about the y-axis and then rotate it clockwise $90°$.

   (a) Find its standard matrix $[T] = [T(\mathbf{e}_1), T(\mathbf{e}_2)]$. Show your work.

   (b) Find $T(\mathbf{x})$ of the vector $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

3. (25 pts) Matrix $A = \begin{bmatrix} 1 & 2 & * & 3 & 22 \\ -3 & 0 & * & 1 & 2 \\ 0 & 0 & 0 & 1 & 5 \\ 1 & 2 & * & -1 & 2 \\ -2 & 2 & * & 5 & 29 \end{bmatrix}$ is row-equivalent to $B = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ -1 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find the reduced row echelon form, rref(A) of A.

(b) Find a largest set of linearly independent column vectors of A.

(c) Can column 5 of A be a linear combination of column 1 through column 4? If not, explain why. If yes, find the combination.

(d) Find the solution to $A\mathbf{x} = \mathbf{0}$ and express the solution in vector form.

4. (13 pts) Use elementary row reduction to find the inverse A^{-1} if $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. You must show each step to receive credit.

5. (10 pts) Find the LU-factorization of matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 1 & 5 \end{bmatrix}$.

6. (12 pts) Determine if vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}$ is in the span $\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$. If yes, write $\mathbf{v}$ as a linear combination of the spaning vectors. (**Consider to use a calculator to save time.**)

7. (10 pts) Set up a balance equation only for the following chemical reaction when sugar is oxidized to produce carbon dioxide and water:

$$C_{12}H_{22}O_{11} + O_2 \rightarrow CO_2 + H_2O$$

Do not solve for the solution.

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2 Bonus Points: True or False: The Nebraska capital was used to be in Omaha. (... The End)