

Print Your Name Legibly: \_\_\_\_\_ Score: \_\_\_\_\_

**Instructions:** You must show supporting work to receive full and partial credits. No textbook, lecture notes, or formula sheets are allowed.

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1. (15 pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. (Recall: an  $m \times n$  matrix is one that has  $m$  rows and  $n$  columns.)
  - (a) T or F? For matrixes  $A, B, C$ , if  $AB = AC$  then  $B = C$ .
  - (b) T or F? If a square matrix  $A$  is row-equivalent to the identity matrix, then it must be invertible.
  - (c) T or F? Every transformation is a linear transformation.
  - (d) T or F? If the columns of an  $n \times n$  matrix  $A$  are linearly dependent, then they span the space  $\mathbb{R}^n$ .
  - (e) T or F? Every system of two equations in three unknowns has infinitely many solutions.
2. (15 pts) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation that first reflect a vector about the  $y$ -axis and then rotate it clockwise  $90^\circ$ .
  - (a) Find its standard matrix  $[T] = [T(\mathbf{e}_1), T(\mathbf{e}_2)]$ . Show your work.

(b) Find  $T(\mathbf{x})$  of the vector  $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ .

3. (25 pts) Matrix  $A = \begin{bmatrix} 1 & 2 & * & 3 & 22 \\ -3 & 0 & * & 1 & 2 \\ 0 & 0 & 0 & 1 & 5 \\ 1 & 2 & * & -1 & 2 \\ -2 & 2 & * & 5 & 29 \end{bmatrix}$  is row-equivalent to  $B = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ -1 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ .

(a) Find the reduced row echelon form,  $\text{rref}(A)$  of  $A$ .

(b) Find a largest set of linearly independent column vectors of  $A$ .

(c) Can column 5 of  $A$  be a linear combination of column 1 through column 4? If not, explain why. If yes, find the combination.

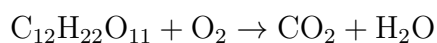
(d) Find the solution to  $A\mathbf{x} = \mathbf{0}$  and express the solution in vector form.

4. (13 pts) Use elementary row reduction to find the inverse  $A^{-1}$  if  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ . You must show each step to receive credit.

5. (10 pts) Find the  $LU$ -factorization of matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 1 & 5 \end{bmatrix}$ .

6. (12 pts) Determine if vector  $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}$  is in the span  $\text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$ . If yes, write  $\mathbf{v}$  as a linear combination of the spanning vectors. (**Consider to use a calculator to save time.**)

7. (10 pts) Set up a balance equation only for the following chemical reaction when sugar is oxidized to produce carbon dioxide and water:



**Do not solve for the solution.**