Math 314 Fall 2016 Final Exam

Print Your Name Legibly:______________________________ **Score:** ____________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, or formula sheets allowed.

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1. (20 pts) For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

   (a) T or F? All elementary matrixes are invertible.

   (b) T or F? Every orthogonal matrix is symmetric.

   (c) T or F? If A is an $m \times n$ with $m < n$, then the dimension of its column space is greater than the dimension of its row space.

   (d) T or F? The null space null(A) of a matrix A is orthogonal to the column space of A^T.

   (e) T or F? The singular values of a matrix can be zeros.

2. (15 pts) Let $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. Use elementary row reduction to find the inverse A^{-1}. (You must show work to receive credit.)

3. (15 pts) Let $\mathcal{B} = \{1+t, 2t+t^2, 1-t^2\}$ be a basis of $\mathbb{P}_2$ and $\mathcal{C} = \{1, t, t^2\}$ be its standard basis. Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, $P_{\mathcal{C}\leftarrow\mathcal{B}}$.

4. (20 pts) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

(a) Find the characteristic equation for A.

(b) Find all eigenvalues of A

5. (20 pts) Use the Gram-Schmidt process to find an orthogonal basis for the subspace $W = \text{Span}\{\begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}\}$.

6. (15 pts) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

(a) Find the distance between $\mathbf{u}$ and $\mathbf{v}$.

(b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

7. (20 pts) Let $W = \text{span}\{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}\}$. Find a basis for the orthogonal complement $W^{\perp}$ of W.

8. (20 pts) For matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 2 \\ -2 & 0 & 0 \end{bmatrix}$, you are given the fact that the eigenvalues are $0,\ 1,\ 2$.

(a) Find all eigenspaces of the eigenvalues.

(b) Can A be diagonalized? If yes, find the diagonalization.

9. (20 pts) For $A = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, you are given the fact that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ are the eigenvectors.

(a) What are the eigenvalues?

(b) Use the information given and (a) above, write a spectral decomposition for A.

10. (15 pts) The singular value decomposition of a 2×3 matrix is given as

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4/5 & 3/5 & 0 \\ 0 & 0 & 1 \\ -3/5 & 4/5 & 0 \end{bmatrix}$$

(a) What are the eigenvalues of the product $A^T A$?

(b) What are the eigenvectors of $A^T A$?

11. (20 pts) Find a singular value decomposition of $A = \begin{bmatrix} 3 & 0 \\ 0 & 0 \\ 4 & 0 \end{bmatrix}$.

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2 Bonus Points: True or False: Huskers Women's Volleyball Team was NCAA's 2015 National Champions.