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Do Not Copy Problem Statement. Submit Solutions to Gradescope by Their Problem Numbers, One Problem per Page. Do not use pencils. There Are Six Problems Plus a Bonus Question on Page 6. Submission Deadline: 10:45 AM. Read the “Rules and Format for Exams” before You Start.

1(15pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

- (a) T or F? If $P_{\mathcal{C} \leftarrow \mathcal{B}}$ is a change-of-coordinate matrix, then 1 must be one of its eigenvalues.
- (b) T or F? The dimension of the row space of a matrix is the number of columns of the matrix minus the matrix’s nullity.
- (c) T or F? An invertible matrix A is always diagonalizable.
- (d) T or F? A triangular matrix can have complex eigenvalues.
- (e) T or F? The first column of the matrix A of a linear transformation $T : V \rightarrow W$ must be the coordinate with respect to a basis of W for the image under T of a basis vector of V .

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2(15pts) Let $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ -2 & 4 & 0 & -2 \\ 2 & -2 & 3 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix}$.

- (a) Find a basis of the row space of A that consists of row vectors of A .
- (b) Write all non-basis rows of A as linear combinations of the basis row vectors from (a).

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3(15pts) Let $A = \begin{bmatrix} 11 & -5 & -15 \\ -2 & 2 & 3 \\ 8 & -4 & -11 \end{bmatrix}$.

(a) Verify that $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ are eigenvectors of A .

(b) Is A diagonalizable? Justify your answer and find the diagonalization $A = PDP^{-1}$ if your answer is yes.

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4(15pts) Let $\mathcal{B} = \{1 + 2t, 2 + t - 5t^2, 1 - 3t^2\}$ and $\mathcal{C} = \{2t + 3t^2, 1 - 2t + t^2, 3 - 5t + 4t^2\}$ for $\mathbb{P}_2$.

- (a) Verify that both $\mathcal{B}$ and $\mathcal{C}$ are bases for $\mathbb{P}_2$. (*Hint:* use the coordinate isomorphism $[\cdot]_{\mathcal{E}}$.)
- (b) Find the change-of-basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.
- (c) Write $p = 1 + 2t$ as a linear combination of the polynomials of $\mathcal{C}$. Explain why it is related to the change-of-basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$.

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5(20pts) Let $A = \begin{bmatrix} 5 & 0 & -5 \\ 0 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$.

- (a) Find the characteristic equation of A .
- (b) Find all eigenvalues of the matrix.
- (c) Find the eigenspace E_λ only for the **real number** eigenvalue λ of A .
- (d) Verify that $\vec{v} = \begin{bmatrix} 3 - i \\ 0 \\ 2 \end{bmatrix}$ is a complex-valued eigenvector of A for eigenvalue $2 - i$.

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6(20pts) Let $\mathcal{M} = \left\{ A = \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} : a, b, c \text{ any real numbers.} \right\}$ be the vector subspace of 2-by-2 matrices. Define $T : \mathcal{M} \rightarrow \mathbb{R}^2$ by

$$T(A) = \begin{bmatrix} a - 2b + c \\ b + 2c \end{bmatrix}.$$

- (a) Demonstrate that T is a linear transformation.
- (b) Find a standard basis $\mathcal{E}$ for $\mathcal{M}$.
- (c) Find the matrix of transformation for T from the coordinate space of $\mathcal{M}$ to the coordinate space of $\mathbb{R}^2$ respect to their standard bases.

2 pt Bonus Question: The first capital city of Nebraska was: (a) Lincoln (b) Grand Island (c) Omaha
(d) None of the above. (... *The End*)