Math 314 Spring 2021 **Exam 2**

Print Your Name Legibly: ______________________________ NUID: ________________

Do Not Copy Problem Statement. Submit Solutions to Gradescope by Their Problem Numbers, One Problem per Page. Do not use pencils. There Are Six Problems Plus a Bonus Question on Page 6. Submission Deadline: 10:45 AM. Read the "Rules and Format for Exams" before You Start.

1(15pts) True/False. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer.

(a) T or F? If $P_{\mathcal{C}\leftarrow\mathcal{B}}$ is a change-of-coordinate matrix, then 1 must be one of its eigenvalues.

(b) T or F? The dimension of the row space of a matrix is the number of columns of the matrix minus the matrix's nullity.

(c) T or F? An invertible matrix A is always diagonalizable.

(d) T or F? A triangular matrix can have complex eigenvalues.

(e) T or F? The first column of the matrix A of a linear transformation $T : V \to W$ must be the coordinate with respect to a basis of W for the image under T of a basis vector of V.

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2(15pts) Let $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ -2 & 4 & 0 & -2 \\ 2 & -2 & 3 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix}$.

(a) Find a basis of the row space of A that consists of row vectors of A.

(b) Write all non-basis rows of A as linear combinations of the basis row vectors from (a).

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3(15pts) Let $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ 3 & -3 & -2 \end{bmatrix}$.

(a) Verify that $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ are eigenvectors of A.

(b) Is A diagonalizable? Justify your answer and find the diagonalization $A = P^{-1}DP$ if your answer is yes.

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4(15pts) Let $\mathcal{B} = \{1 + 2t, 2 + t - 5t^2, 1 - 3t^2\}$ and $\mathcal{C} = \{2t + 3t^2, 1 - 2t + t^2, 3 - 5t + 4t^2\}$ for $\mathbb{P}_2$.

(a) Verify that both $\mathcal{B}$ and $\mathcal{C}$ are bases for $\mathbb{P}_2$. (*Hint:* use the coordinate isomorphism $[\,\cdot\,]_{\mathcal{E}}$.)

(b) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Write $p = 1 + 2t$ as a linear combination of the polynomials of $\mathcal{C}$. Explain why it is related to the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$.

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5(20pts) Let $A = \begin{bmatrix} 5 & 0 & -5 \\ 0 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$.

(a) Find the characteristic equation of A.

(b) Find all eigenvalues of the matrix.

(c) Find the eigenspace E_λ only for the **real number** eigenvalue λ of A.

(d) Verify that $\vec{v} = \begin{bmatrix} 2+i \\ 0 \\ 1+i \end{bmatrix}$ is a complex-valued eigenvector of A for eigenvalue $2-i$.

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6(20pts) Let $\mathcal{M} = \left\{ A = \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} : \ a, b, c \text{ any real numbers.} \right\}$ be the vector subspace of 2-by-2 matrices. Define $T : \mathcal{M} \to \mathbb{R}^2$ by

$$T(A) = \begin{bmatrix} a - 2b + c \\ b + 2c \end{bmatrix}.$$

(a) Demonstrate that T is a linear transformation.

(b) Find a standard basis $\mathcal{E}$ for $\mathcal{M}$.

(c) Find the matrix of transformation for T from the coordinate space of $\mathcal{M}$ to the coordinate space of $\mathbb{R}^2$ respect to their standard bases.

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2 pt Bonus Question: The first capital city of Nebraska was: (a) Lincoln (b) Grand Island (c) Omaha (d) None of the above. (... *The End*)