NAME: ______________________

INSTRUCTIONS: No references, ***no calculators****, no cellphones. Please* ***clearly organize your solutions*** *and* ***emphasize the answers****. You must* ***show all details of your work*** *to receive credit. If unable to solve, you may earn partial credit by outlining which solution strategy you would attempt.*

The exam has a total of 60 points (= 100%).

1. (5pts) Suppose P and Q are $n \times n$ orthogonal matrices. Expand and simplify the following expression:

$$\left((P+Q)(P-Q)^T\right)^T$$

2. (7pts) Complete the missing entries in the probability matrix P and find the steady state vector:

$$P = \begin{bmatrix} * & 3/4 \\ 1/2 & * \end{bmatrix}$$

3. (10pts) Let $W = \left\{ \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} : x - y = 0 \quad \text{and} \quad w - y - z = 0 \right\}$. Find a basis for $W^{\perp}$.

4. (10pts) Apply the Gram-Schmidt algorithm to vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ in order to construct an orthonormal basis for $\mathbb{R}^3$.

5. (8pts) Find the missing entries to make Q an *orthogonal* matrix

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & * \\ 0 & \frac{1}{\sqrt{3}} & * \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & * \end{bmatrix}$$

6. (7pts) Let W be a subspace of $\mathbb{R}^n$. Show that if $\mathbf{v}$ belongs to $W \cap W^{\perp}$ (to both W and $W^{\perp}$), then $\mathbf{v}$ must necessarily be the zero vector.

7. (5pts) Prove or give a counter-example to the statement: the set S of all 2×2 **non-invertible diagonal** matrices, forms a subspace of M_{22}.

   (Recall that M_{22} is the vector space of all 2×2 matrices with the usual matrix addition and scalar multiplication).

8. (8pts) Let W be a subspace of $\mathbb{R}^4$. Suppose its orthogonal complement is given by $W^\perp = \text{span}\left\{ \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. Find the orthogonal decomposition of $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ with respect to W.