NAME: ____________________

INSTRUCTIONS: No references, ***no calculators****, no cellphones. Please* ***clearly organize your solutions*** *and* ***emphasize the answers****. You must* ***show all details of your work*** *to receive credit. When unable to solve, you may earn partial credit by outlining which solution strategy you would attempt.*

The exam has a total of 65 points (= 100%).

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1. Let $A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 1 & 0 & -1 & -1 \\ -1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (a) (5pts) Find the characteristic polynomial of A
   (you don't have to multiply it out into the standard form for a polynomial).

   (b) (4pts) Find the eigenvalues of A and their algebraic multiplicities.

1. (cont.) The same matrix: $A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 1 & 0 & -1 & -1 \\ -1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (c) (10pts) Find a basis for **ONE** of the eigenspaces of A.

2. (3pts) Assume that A and B are $n \times n$ matrices, with $\det A = 3$ and $\det B = -2$. What is the determinant of $B^{-1}A$?

3. (8pts) Find all values of k for which matrix $A = \begin{bmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{bmatrix}$ is invertible.

4. (2pt) Can an eigenspace have dimension 0? Explain (concisely!).

5. (5pts) Prove that if A is an $n \times n$ diagonalizable matrix with only one eigenvalue λ then A is of the form $A = \lambda I$. Your answer should be logical and complete.
(But please refrain from writing essays).

6. (4pt) Matrix

$$\begin{bmatrix} -6 & 0 & 0 & 0 & -6 & 0 \\ -3 & 0 & -2 & 2 & -3 & 2 \\ 13 & -5 & -2 & -2 & 6 & 4 \\ -17 & 6 & 6 & 7 & -6 & -6 \\ -9 & 5 & 5 & 10 & 2 & -5 \\ 2 & 2 & 3 & 4 & 6 & -1 \end{bmatrix}$$

has eigenvector $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. What is the corresponding eigenvalue?

7. Given the following similarity relation $P^{-1}AP = D$.

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} -1 & -1 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}}^{A} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (4pts) What are the eigenvalues of A and their algebraic multiplicities?

(b) (4pts) Give a basis for each eigenspace of A.

7. (cont.) The same similarity relation: $P^{-1}AP = D$:

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} -1 & -1 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}}^{A} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) (8pts) Let $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$. Find a formula for $A^n\mathbf{v}$ ($n \geq 0$). You must simplify your answer as much as possible.

8. (8pts) Set up the system of linear equations $A\mathbf{x} = \mathbf{b}$ (you must specify what A and $\mathbf{b}$ are) for the unknown vector $\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T$ which describes the traffic pattern in the freeway network shown in the figure.

You do NOT need to solve the system.