

Name: _____

Score: _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets are allowed.

1(15 pts) Solve the initial value problem

$$y'' + 4y' + 5 = 0, \quad y(0) = -2, \quad y'(0) = 1.$$

2(20 pts) Determine the **form** only for a particular solution for each of the equations. (You only need to solve the homogeneous equation once.)

(a) $y'' + 2y' + 2y = 1 + t$.

(b) $y'' + 2y' + 2y = e^{-t} \sin t$.

(c) $y'' + 2y' + 2y = 1 + te^{3t}$.

3(25 pts) Use the method of undetermined coefficients to find a particular solution to the equation

$$y'' - y = t^2$$

and then find a general solution.

4(15 pts) If the characteristic equation for a 5th-order homogeneous and linear equation with constant coefficients

$$a_0 y^{(5)} + a_1 y^{(4)} + \cdots + a_4 y' + a_5 y = 0$$

has 0 as a double root, and $3, -3 \pm i$ are the other roots, what is the general solution for the equation?

5(25 pts) Use the method of variation of parameters to find a particular solution to the equation

$$y'' - 2y' + y = \frac{e^t}{t}.$$

(Note: you can use the fact that $y_1(t) = e^t, y_2(t) = te^t$ form a fundamental set of solutions to the homogeneous equation.)

END