Math 221 Sec.4 **Test I** **Feb. 8, 2001**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15 pts) Verify that $\ln y + xy = 5$ defines an implicit solution to the equation $\frac{dy}{dx} = -y^2/(1+xy)$.

2(18 pts) Find the solution to the initial value problem $\frac{dy}{dx} = \frac{x}{1+y}$, $y(0) = 1$.

3(18 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = (x+1)(y^4+1), y(0) = 0$ at these points $x = 0, 0.25, 0.5$.
(b) Plot your approximating solution in the xy plane.

4(16 pts) Find the general solution to the equation $y' - \frac{1}{(1+t)}y = 1$.

5(18 pts) Consider the equation

$$\frac{dy}{dt} = f(y) = (1-y)(y-k),$$

where k is a parameter.

(a) Sketch a phase line for the equation for some value of k between 0 and 1. Classify each equilibrium point as sink, source, or node.
(b) Sketch a few typical solutions qualitatively with y v.s. the time t, for the same value of k you used in (a) above.
(c) For what value of k does the equation has only one equilibrium point? Classify that point as sink, source, or node.

6(15 pts) Blood carries a drug into an organ at a rate of 3 cm^3/sec and leaves at the same rate. The organ has a liquid volume of 125 cm^3 which has no trace of the drug initially. If the concentration of the drug in the blood entering the organ is 0.2 g/cm^3, write down an initial value problem for the amount of drug in gram at any time t. (*Remark*: Don't think about by-passing differential equations to solve this problem.)

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