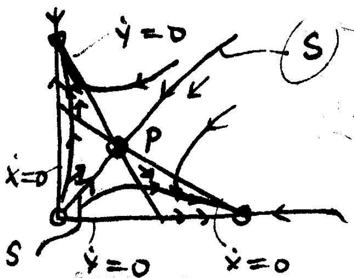
8 (18 pts) (a) $|A-\lambda I| = \begin{vmatrix} 3-\lambda & -5 \\ 2 & -4-\lambda \end{vmatrix} = \lambda^2+\lambda-2 = (\lambda+2)(\lambda-1)=0,\ \lambda_1 = \cancel{-2},\ \lambda_2=1$

(b) For $\lambda_2=1$, $(A-\lambda_2 I)\vec{u} = \vec{0}$. $\begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $2x_1-5x_2=0$, $x_2=2, x_1=5$,

$\Rightarrow \vec{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

9



(a)

(b) - They co-exist only if the populations start out on the separatrix S or the equilibrium state P.

- If the population start out below S, y-species will eventually die out.
- If they start out above S, the x-species will eventually die out.

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10 (18 pts) (a) $\dfrac{1}{s^2-2s-3} = \dfrac{1}{(s-3)(s+1)} = \dfrac{A}{s-3} + \dfrac{B}{s+1} = \dfrac{A(s+1)+B(s-3)}{(s-3)(s+1)}$, $\quad A+B=0$, $A-3B=1$

$\Rightarrow -4B=1,\ B=-\frac{1}{4},\ A=\frac{1}{4},\ \mathcal{L}^{-1}\left\{\dfrac{1}{s^2-2s-3}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\dfrac{1}{s-3}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\dfrac{1}{s+1}\right\} = \frac{1}{4}e^{3t}-\frac{1}{4}e^{-t}$

(b) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-2s+3}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^2+2}\right\} = \dfrac{1}{\sqrt{2}}\mathcal{L}^{-1}\left\{\dfrac{\sqrt{2}}{(s-1)^2+(\sqrt{2})^2}\right\} = \dfrac{1}{\sqrt{2}}e^{t}\sin\sqrt{2}t$

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11 (16 pts) $\mathcal{L}\{y'''\} - 2\mathcal{L}\{y\} = 2\mathcal{L}\{e^{2t}\cos 3t\} = 2\dfrac{s-2}{(s-2)^2+9}$

$\Rightarrow s^3\mathcal{L}\{y\} - s^2y(0) - sy'(0) - y''(0) - 2\mathcal{L}\{y\} = \dfrac{2(s-2)}{(s-2)^2+9}$

$(s^3-2)\mathcal{L}\{y\} = 2s^2 + \dfrac{2(s-2)}{(s-2)^2+9}$. $\quad \mathcal{L}\{y\} = \dfrac{2s^2}{s^3-2} + \dfrac{2(s-2)}{(s^3-2)((s-2)^2+9)}$

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12 (18pts) $y'-y=1,\ y(0)=2,\quad \mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{1\} = \frac{1}{s}$

$s\mathcal{L}\{y\} - y(0) - \mathcal{L}\{y\} = (s-1)\mathcal{L}\{y\} - 2 = \frac{1}{s},\quad \mathcal{L}\{y\} = \dfrac{2}{s-1} + \dfrac{1}{s(s-1)}$

$\dfrac{1}{s(s-1)} = \dfrac{1}{s-1} - \dfrac{1}{s} \Rightarrow \mathcal{L}\{y\} = \dfrac{3}{s-1} - \dfrac{1}{s},\quad y = 3\mathcal{L}^{-1}\left\{\dfrac{1}{s-1}\right\} - \mathcal{L}^{-1}\left\{\dfrac{1}{s}\right\} = 3e^{t}-1$

End.