Math221 Final Exam Solutions

1 (16pts) $\frac{1}{\cos x}\frac{dy}{dx} = \frac{1}{y+1}$, $(y+1)dy = \cos x\,dx$, $\int (y+1)dy = \int \cos x\,dx$,

$\frac{(y+1)^2}{2} = \sin x + c$, $\Rightarrow (y+1)^2 = 2\sin x + c$, $y = -1 \pm \sqrt{2\sin x + c}$

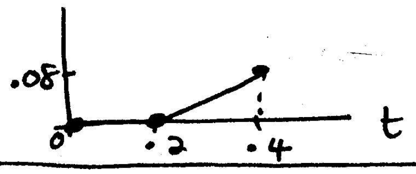
2 (16pts) $y' + \frac{2}{t}y = 1$, $y(1) = 2$, $p(t) = \frac{2}{t}$, $\mu(t) = e^{\int p(t)dt} = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$t^2y' + 2ty = t^2$; $(t^2y)' = t^2$, $t^2y = \int t^2 dt = \frac{1}{3}t^3 + C$, $y = \frac{1}{3}t + \frac{c}{t^2}$.

$2 = y(1) = \frac{1}{3} + C$, $C = 2 - \frac{1}{3} = \frac{5}{3}$, $y(t) = \frac{1}{3}t + \frac{5}{3}\frac{1}{t^2}$

3 (18pts) (a) $t_0 = 0$, $y_0 = 0$, $y_{k+1} = y_k + hf(t_k, y_k) = y_k + .2(2t_k - y_k^2)$

k	t_k	y_k	$f(t_k, y_k)$
0	0	0	0
1	.2	0	.4
2	.4	.08	



4 (15pts) $y'' + 2y' + y = 0$, $r^2 + 2r + 1 = (r+1)^2 = 0$, $r_{1,2} = -1$.

$y(t) = c_1e^{-t} + c_2te^{-t}$

5 (16pts) $y' - y = t^2 - (t-1)\sin 3t + 2e^t$, $y' - y = 0$, $r - 1 = 0$, $r = 1$, $y_1(t) = e^t$

$g_1(t) = t^2$, $Y_1(t) = t^s(At^2 + Bt + C)$ w/ $s = 0$ since $t^k \neq y_1(t) = e^t$

$g_2(t) = -(t-1)\sin 3t$, $Y_2(t) = t^s[(D_1t + E_1)\cos 3t + (D_2t + E_2)\sin 3t]$ w/ $s = 0$ since none of the terms is of the form $y_1(t)$.

$g_3(t) = 2e^t$, $Y_3(t) = t^sFe^t$ w/ $s = 1$ since $e^t = y_1(t)$ for $s = 0$ and $Fte^t \neq cy_1(t)$,

Finally, $Y(t) = Y_1(t) + Y_2(t) + Y_3(t) = At^2 + Bt + C + (D_1t + E_1)\cos 3t + (D_2t + E_2)\sin 3t + Fte^t$

6 $y' - 3y = 3t$, $y' - 3y = 0 \Rightarrow r - 3 = 0$, $y_1(t) = e^{3t}$,

$g(t) = 3t$, $Y(t) = (At + B)t^s$ w/ $s = 0$. $Y'(t) = A$, $Y' - 3Y = A - 3(At + B)$

$= -3At + A - 3B = 3t$, $-3A = +3$, $A - 3B = 0$ $\Rightarrow A = -1$, $B = -\frac{1}{3}$ $\Rightarrow$

$Y(t) = -t - \frac{1}{3}$

7. (a) $e^{(1+2i)t}\vec{u} = e^t(\cos 2t + i\sin 2t)\begin{pmatrix}1-i\\2\end{pmatrix} = e^t\begin{pmatrix}\cos 2t + \sin 2t + i(\sin 2t - \cos 2t)\\2\cos 2t + i\,2\sin 2t\end{pmatrix}$

$\vec{x}(t) = c_1e^t\begin{pmatrix}\cos 2t + \sin 2t\\2\cos 2t\end{pmatrix} + c_2e^t\begin{pmatrix}\sin 2t - \cos 2t\\2\sin 2t\end{pmatrix}$

(b) $\vec{x}_1(t) = e^{-2t}\begin{pmatrix}-1\\1\end{pmatrix}$

$\vec{x}_2(t) = te^{\lambda t}\vec{u} + e^{\lambda t}\vec{v}$

$= e^{-2t}\begin{pmatrix}-t+2\\t\end{pmatrix}$