Mathematics 221 **Name**
Test 2 Fall 2004

1. (10 pts) Find the equation of the trajectories in the xy plane for the system $x' = 4y, \quad y' = 2x - 2$.

2. (15 pts) Consider the population model governed by the autonomous equation
$$p' = \sqrt{2}\,p - \frac{4p^2}{1+p^2}.$$
   (a) Sketch a graph of the growth rate p' versus the population p, and sketch the phase line.
   (b) Find the equilibrium populations and determine their stability.

3. (10 pts) For the following system, for which values of the constant b is the origin an unstable spiral?
$$\begin{aligned} x' &= x - (b+1)y \\ y' &= -x + y \end{aligned}$$

4. (15 pts) Consider the nonlinear system
$$\begin{aligned} x' &= x(1 - xy), \\ y' &= 1 - x^2 + xy. \end{aligned}$$
   (a) Find all the critical points (equilibrium solutions).
   (b) In the xy plane plot the x-nullcline(s) (vertical nullcline(s)).

5. (10 pts) Showing all your work, find a linear trajectory for the three dimensional system
$$\vec{x}' = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix} \vec{x}.$$

6. (10 pts) Classify the critical point as to type and stability for the system
$$x' = x + 13y, \quad y' = -2x - y.$$

7. (15 pts) A two-dimensional system $\vec{x}' = A\vec{x}$ has eigenpairs
$$-2, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad 1, \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$
   (a) If $\vec{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, find a formula for $y(t)$. (where $\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.
   (b) Sketch a rough, but accurate, phase portrait.

8. (15 pts) Consider the IVP

$$\begin{aligned} x' &= -2x + 2y \\ y' &= 2x - 5y, \\ x(0) &= 3, \quad y(0) = -3. \end{aligned}$$

(a) Use your calculator's graphical solver to plot the solution for $t > 0$ in the xy phase plane. (reproduce on the axes below).

(b) Using your plot in (a), sketch $y(t)$ versus t for $t > 0$.