Mathematics 221 Fall 04
Sample Test 1

1. For which value(s) of the constant b does the DE $y' + 3y = e^{-2t}$ have a solution of the form $y = be^{-2t}$?

2. For the DE in Problem 1 what is the slope of the minitangent in the slope field at the point $(0, 5)$ in the ty plane?

3. Let the function $y = y(t)$ be defined by the integral

$$y(t) = \int_1^t \frac{1}{\sqrt{s+1}} \sin 2s ds.$$

   Find the derivative $y'(t)$ of the function $y(t)$.

4. Find the solution of the IVP $y' = 5ay, \quad y(0) = y_0$, where a and y_0 are constants.

5. Use separation of variables to solve $y' = y^2 \cos \pi t$, $y(0) = 1$.

6. Find the general solution of the DE $y' = (t+y)^2$ by making a substitution $v = t + y$.

7. For the DE $y' = y^2 - e^{-t}$ draw the zero isocline (the set of points where the slope field is zero) in the ty plane.

8. What is the interval of existence of the solution to the IVP

$$(t-3)^3 y' - t(t-2)y = \cos t, \quad y(1) = 0?$$

9. The solution curves to a certain DE are $y = t^2 + C$, where C is an arbitrary constant. Sketch a rough plot of the slope field of the DE in the window $-2 < t < 2, \quad -4 < y < 4$.

10. Consider a "population" model

$$\frac{dp}{dt} = (p - 0.5)(p - 12)^2.$$

   (a) What are the equilibrium populations?

   (b) Draw the phase line and classify any equilibria at asymptotically stable, unstable, or semistable.

   (c) Sketch a graph of the per capita growth rate $\frac{1}{p}\frac{dp}{dt}$ as a function of the population p.

   (d) Draw a rough time series plot (p versus t) of the solution $p = p(t)$ if $p(0) = 2$. In reality, if this were a real population, what would you expect the population to be after a long time?

   (e) For which initial conditions $p(0)$ does the population become extinct?

11. Consider the IVP in Problem 5. Take the stepsize $h = 0.25$ and use the Euler method to approximate $y(0.25)$. Use the modified Euler method to approximate $y(0.25)$. Work this problem by hand, showing your calculations (you may use a calculator to do the arithmetic).

12. A bowling bass of mass m is dropped from the top of a tall building. It experiences both a force due to gravity and a drag force of magnitude $kv^{2.5}$, where v is its velocity and k is a positive constant. Choose a coordinate system with the height y measured positive downward, with $y = 0$ at the top of the building. Write down an IVP for the velocity $v = v(t)$ that governs the motion of the ball. If the building is very high and the ball reaches a limiting velocity, what is that limiting velocity?